

Quiz 5 Monday

- ↪ Convolution / inverse Laplace w/ convolution
- ↪ Nonlinear DE's

= Critical pts on Non-linear System

= Linearize a NL System

- classify the stability

7.2, 7.3^(?)

Recap: Non-linear System of Eqns.

find CP:

7.1

$$x' = \dots = 0$$

$$y' = \dots = 0$$

Find (x, y) that satisfy.

7.2



$$J = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$$

|| $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

Linearised
matrix
A

Ex

$$1) \quad \begin{aligned} x' &= x - y^2 \\ y' &= x - 2y + x^2 \end{aligned}$$

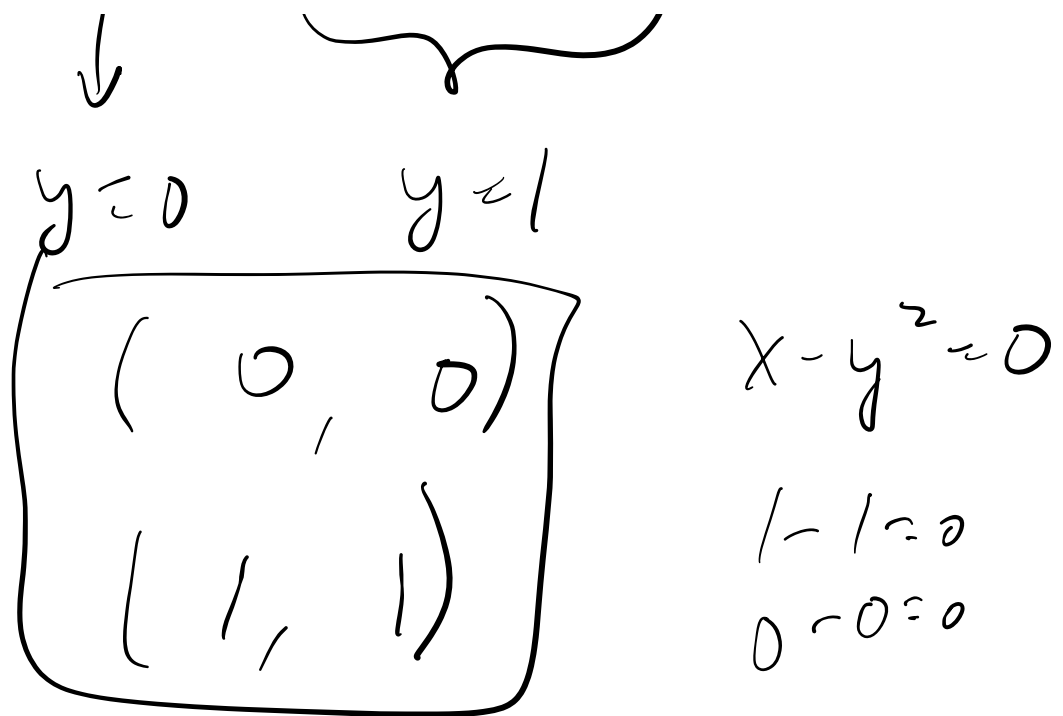
a) Find the CP

$$(0,0) \quad (1,1)$$

$$x = y^2$$

$$y^2 - 2y + y^4 = 0$$

$$y(y^3 + y - 2) = 0$$



b) Linearize around your fixed pts:

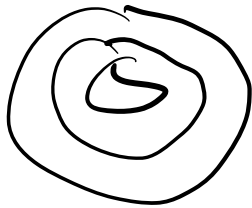
$(0,0)$ Linear system:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix} \Rightarrow 1, -2$$

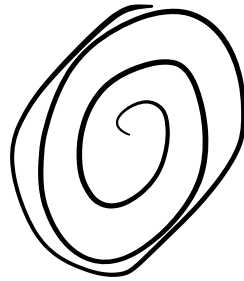
Stability = saddle pt.

(1, 1)

$$A = \begin{bmatrix} 1 & -2 \\ 3 & -2 \end{bmatrix} \Rightarrow \frac{-1 \pm i\sqrt{15}}{2}$$

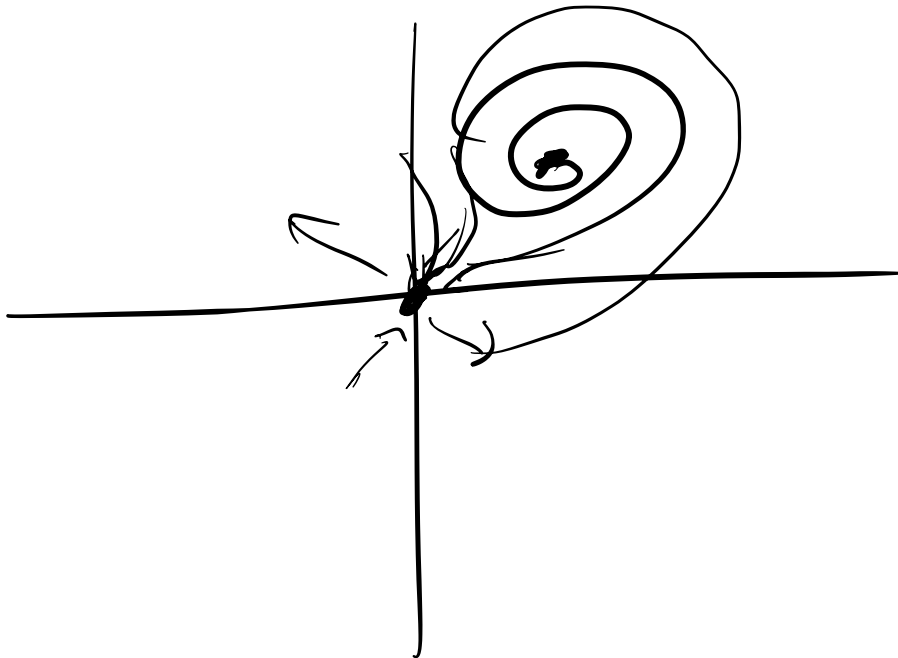


Center
no real part



Stable
spiral
real part

Stable
spiral



Ex.

$$x' = 3 - xy = 0$$

$$y' = x - 3y^3$$

$$3 = xy$$

$$x = \frac{3}{y}$$

$$\frac{3}{y} = 3y^3$$

$$3 = 3y^4$$

$$3 = x(1)$$

$$3 = x(-1)$$

$$y^4 = 1$$

$$y = 1, -1$$

a) critical pts $\rightarrow (3, 1) (-3, -1)$

b) linearize $\begin{bmatrix} -1 & -3 \\ 1 & -9 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 \\ 1 & -9 \end{bmatrix}$

c) stability?

$$\rightarrow -5 \pm \sqrt{13}$$

asymptotically stable

$$-4 \pm 2\sqrt{7}$$

unstable saddle pt.

7.3

$$x' = x(1 - x - y)$$

$$y' = y(0.75 - y - 0.5x)$$

Four critical pts.

3 \rightarrow not stable

at least system
goes extinct

1 \rightarrow symp. stable

both survive.