Apr 17, 2024

Numerical Methods for solving DE

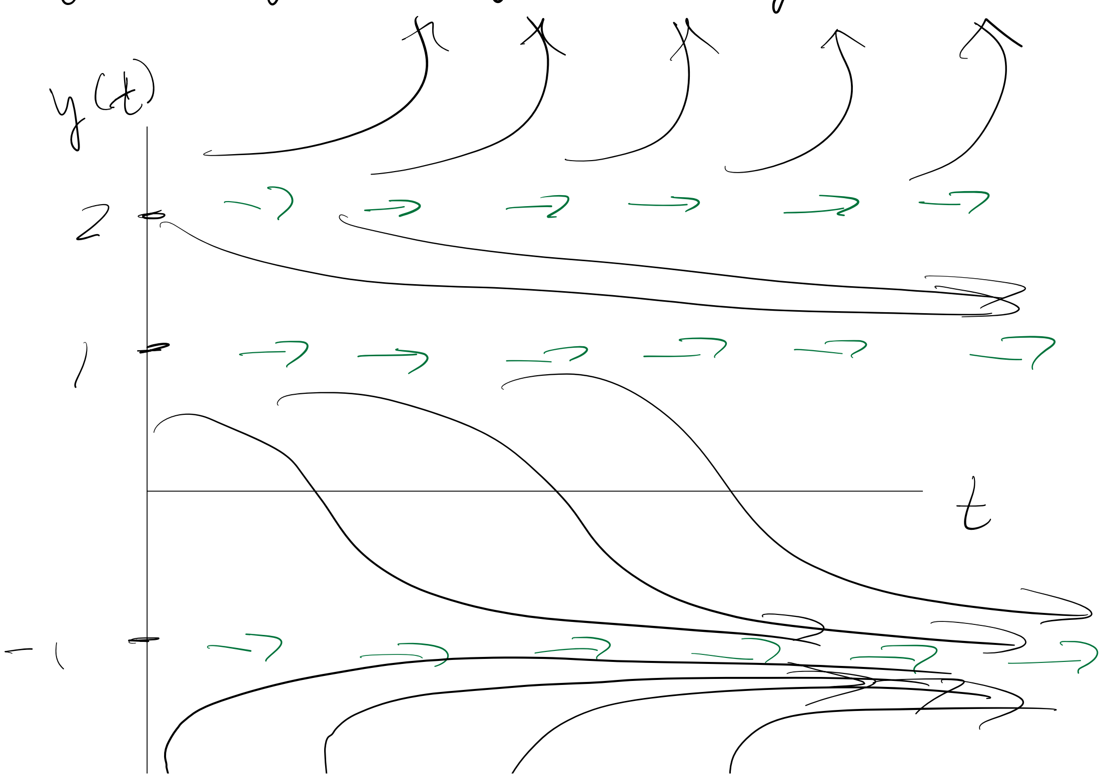
CIOS

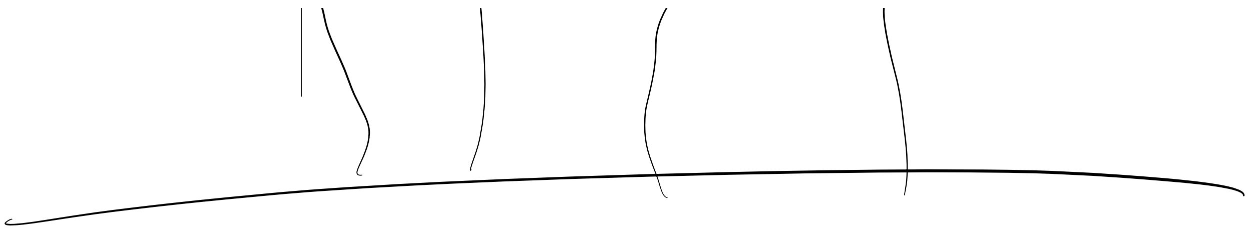
fill it out for +1% grade

8.1 → Euler's Method
8.2 → Estimate Numerical Error } Today
8.3 → Improved Euler, Runge-Kutta } Monday

Direction Field

$$y' = (y-2)(y+1)(1-y)^2$$



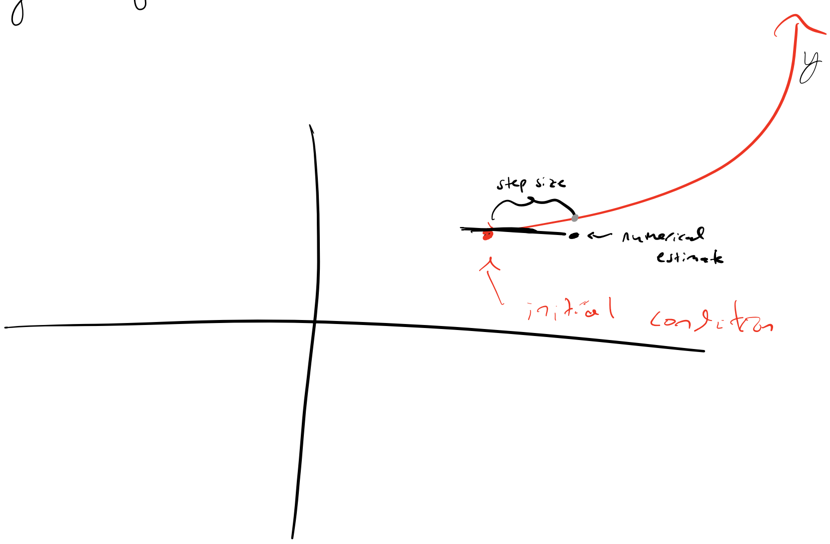


$$\frac{dy}{dt} = f(t, y) \qquad y(t_0) = y_0$$

$t = t_0$

Euler's Method:

$$y = y_0 + f(t_0, y_0)\underbrace{(t - t_0)}_{\Delta h}$$



Solution space

$$y = y_1 + f(t_1, y_1)(t - t_1)$$

$$y_2 = y_1 + f(t_1, y_1)(t_2 - t_1)$$

$$y_3 = y_2 + f(t_2, y_2)(t_3 - t_2)$$

$\vdots$

$t_n - t_{n-1} = h$

usually, $h = .01, .05, .1$

Ex.

$$y' = 2 + 2t - y, \quad y(0) = 1$$

find the approx value at $t = 0.1$ using Euler method w/ $h = 0.05$.

$t_0 = 0 \qquad y_0 = 1$ Remember this formula

$$y_{n+1} = y_n + (2 + 2t_n - y_n) h$$

$$y_1 = y_0 + h(2 + 2t_0 - y_0) = 1 + .05(2 + 2(0) - 1)$$

$$= 1 + 0.05(1) = \underline{1.05}$$

$$y_2 = y_1 + h(2 + 2t_1 - y_1) =$$

$$1.05 + 0.05(2 + 2(.05) - 1.05)$$

$$= 1.05 + .05(2 + .1 - 1.05)$$

$$= 1.05 + .05(1.05) = \boxed{1.1025}$$

8.2

Local truncation error:

$$e_{n+1} = \frac{1}{2} h^2 y''(\bar{t}_n),$$

where $t_n \le \bar{t}_n \le t_{n+1}$

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ex

Given $y' = 2y - 3t$, $y(0) = 1$

a) Estimate the local truncation error for the Euler method in terms of t, the solution, and h.

b) Compute the bound for e_1 at $t = 0.1$ for $h = 0.1$.

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a) Solve

$$y' = 2y - 3t.$$

integrating factor

$$y = \frac{3(2t+1)}{4} + C_1 e^{2t}$$

$$y' = C_2 + 2C_1 e^{2t}$$

$$y'' = 4C_1 e^{2t}$$

$$1 = \frac{3(0+1)}{4} + C_1$$

$$C_1 = \frac{1}{4}$$

$$y'' = e^{2t}$$

$$e_{n+1} = \frac{1}{2} h^2 y''(\bar{t}_n),$$

$$\text{a) } e_{n+1} = \frac{h^2}{2} e^{2t}$$

b) $\frac{(0.1)^2}{2} e^{2(0.1)} \approx .006107$

$$|e_1| \lessapprox .006107$$

Mon – last studio (8.3

Tues – 1-2PM last office hours (Clough 280 Math Lab)

Tues – 6PM – ? Maybe food Final review session

– 6PM-8PM double PLUS session, if you want

Thurs – 2:40 PM

Final Exam

Howey L2