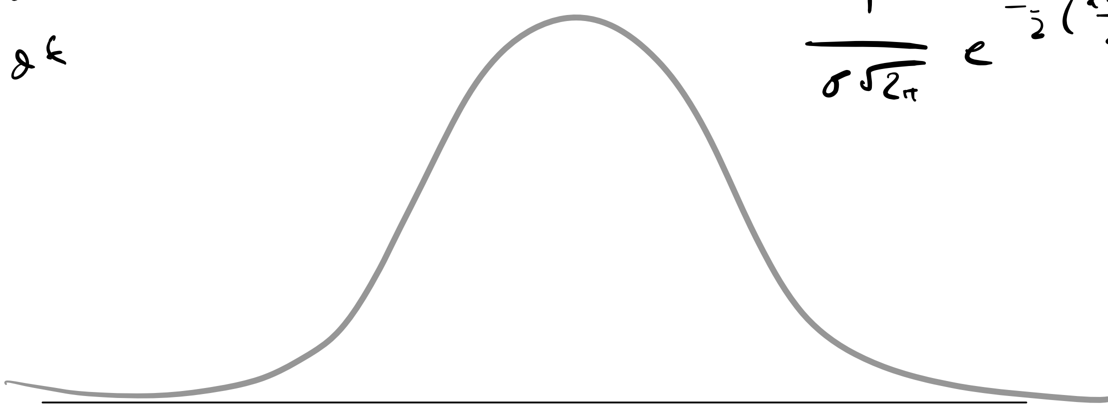
April 3 2024

- Dirac Delta Function (5.7)
- Convolution (5.8)

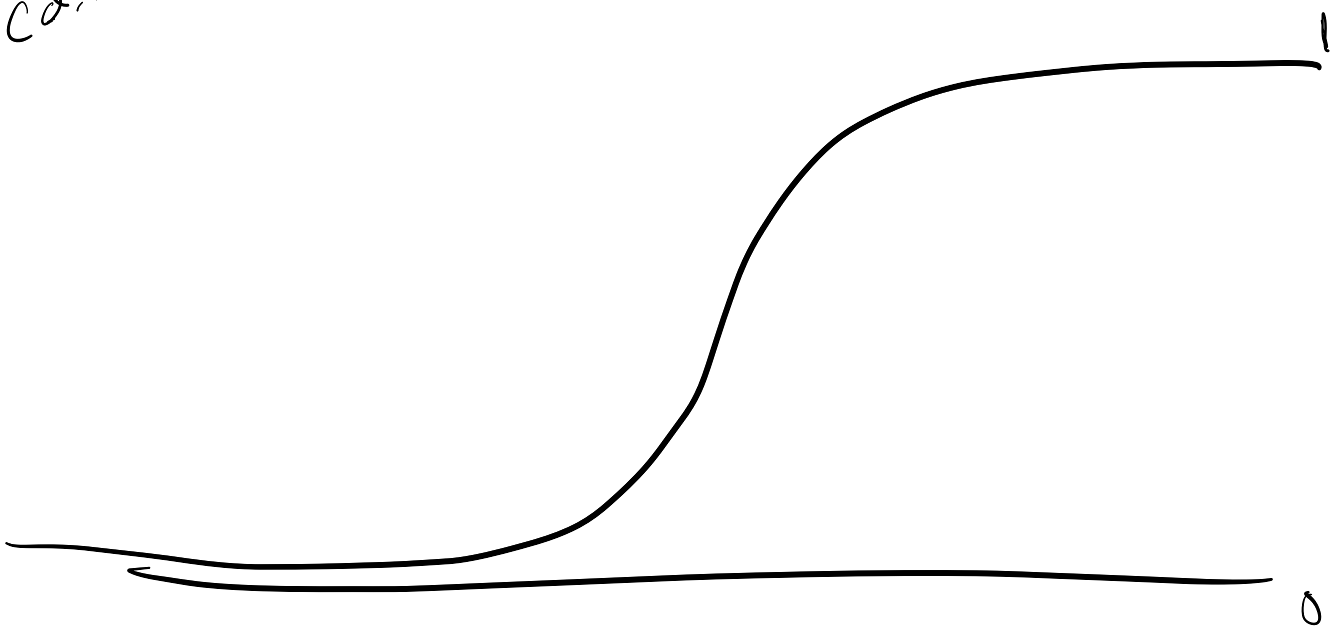
Normal Distribution

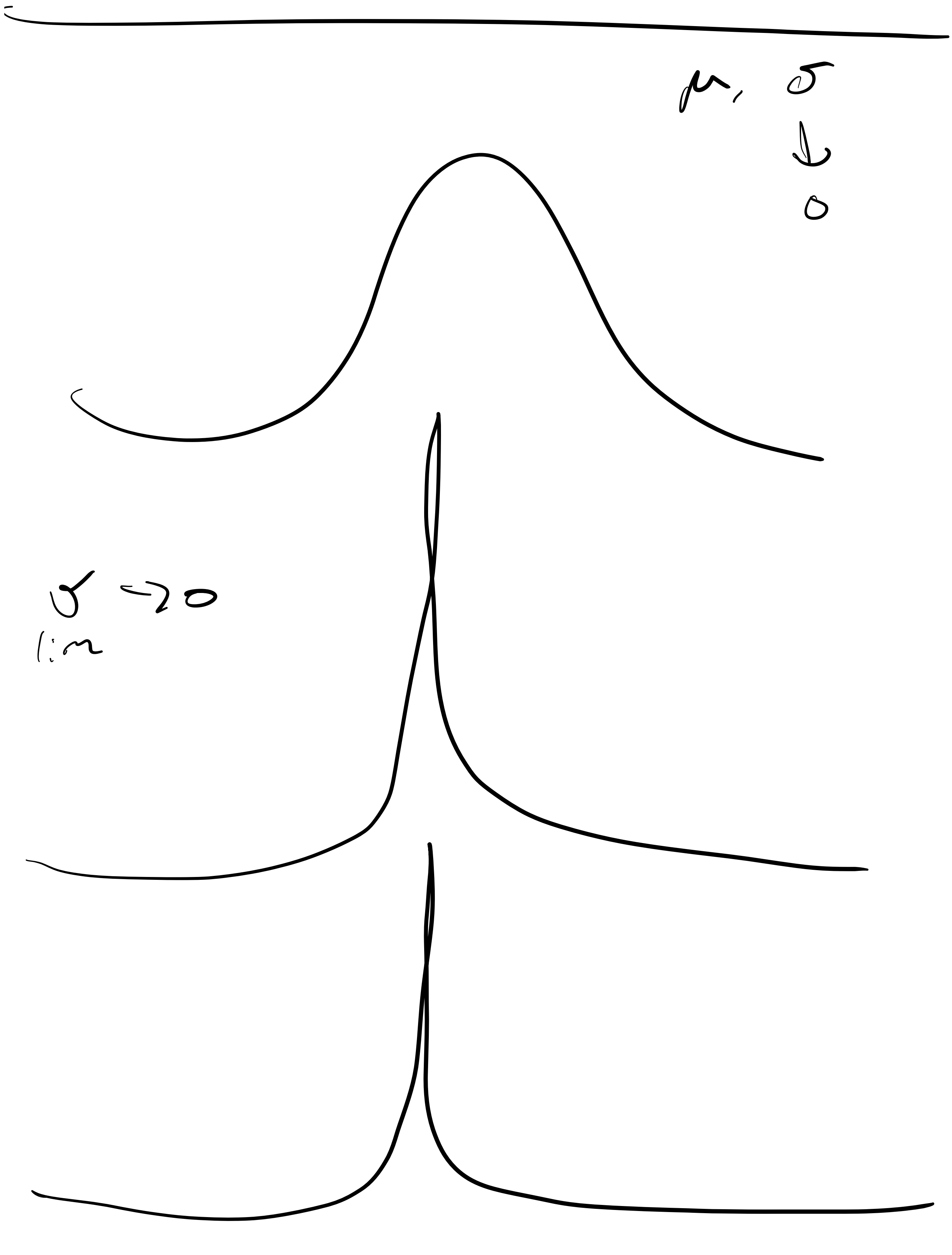
pdf

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



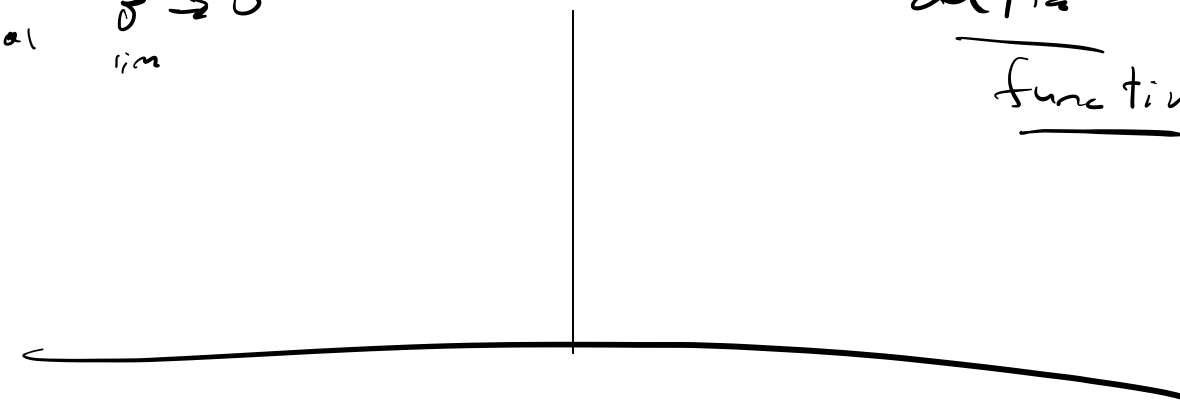
cdf



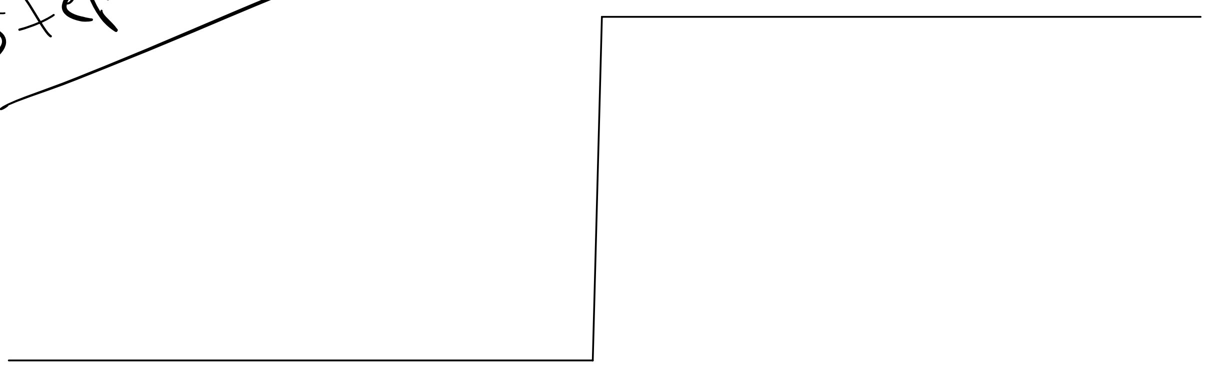

μ, σ
$\downarrow$
0
$\sigma \to 0$
lim

pdf of Normal dist
at $\lim \sigma \to 0$

delta function



c.d.f.
step fn.



delta def.

$$\begin{cases} 0 & t \neq 0 \\ \text{undefined} & t = 0 \end{cases}$$

int (delta) = 1

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}$$

bc $\int_0^\infty e^{-st}\,\delta(t-a)\,dt$

Ex. laplace

$$2\,\delta(t-4)$$

$$= 2e^{-4s}$$

Convolution

imagine you see $H(s)$:

$$H(s) = F(s)G(s)$$

$$\downarrow \qquad \downarrow \qquad \text{inv laplace}$$

$$f(t) \qquad g(t)$$

The convolution integral:

$$(f * g) = \int_0^t f(t-\tau)\,g(\tau)\,d\tau$$

$$\shortparallel$$

$$(g * f)$$

Ex.

$f * g(t) =$

$$\int_0^t f(t-\tau)\, g(\tau)\, d\tau$$

$f(t) = e^{3t}$ $\quad g(t) = e^{7t}$

$g(t-\tau) = e^{7(t-\tau)}$

$$\int_0^t e^{7t} e^{-7\tau} e^{3\tau}\, d\tau$$

$$e^{7t}\int_0^t e^{-4\tau}\,d\tau$$

$$= -\frac{1}{4}e^{3t} + \frac{1}{4}e^{7t}$$

Inv Laplace of this using convolution:

$$\frac{1}{(s+5)^6(s^2+9)}$$

$h(t) = ?$

$H(s) = F(s)\,G(s)$

$F(s) = \frac{1}{(s+5)^6}$ $\quad G(s) = \frac{1}{s^2+9}$

$$\frac{1}{5!}\frac{5!}{(s+5)^6}$$

$$\frac{1}{s^2+3^2} = \frac{3}{3(s^2+3^2)}$$

$$\frac{1}{3}\sin(3t)$$

$$\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$$

$$= \frac{1}{120} t^5 e^{-5t}$$

$$f * g = \int_0^t f(t-\tau)\, g(\tau)\, d\tau$$

$$= \frac{1}{360}\int_0^t (t-\tau)^5 e^{-5(t-\tau)} \sin(3\tau)\, d\tau$$

Side comment

Input Signal

2 10 4

Impulse R

1 5

		10	50	20
+	2	10	4	
	2	20	54	20