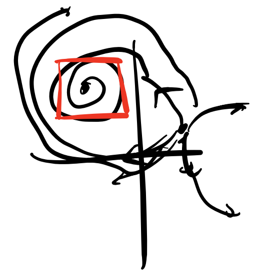
Apr 8, 2024

Non-linear DE



Ch 7 → Linearizing around a certain point

Ch 8 → Numerical Methods

Euler's Method

Improved Euler's Method

Runge-Kutta $4^{th}/5^{th}$

7.1 → Critical points

7.2 → linearizing

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7.1

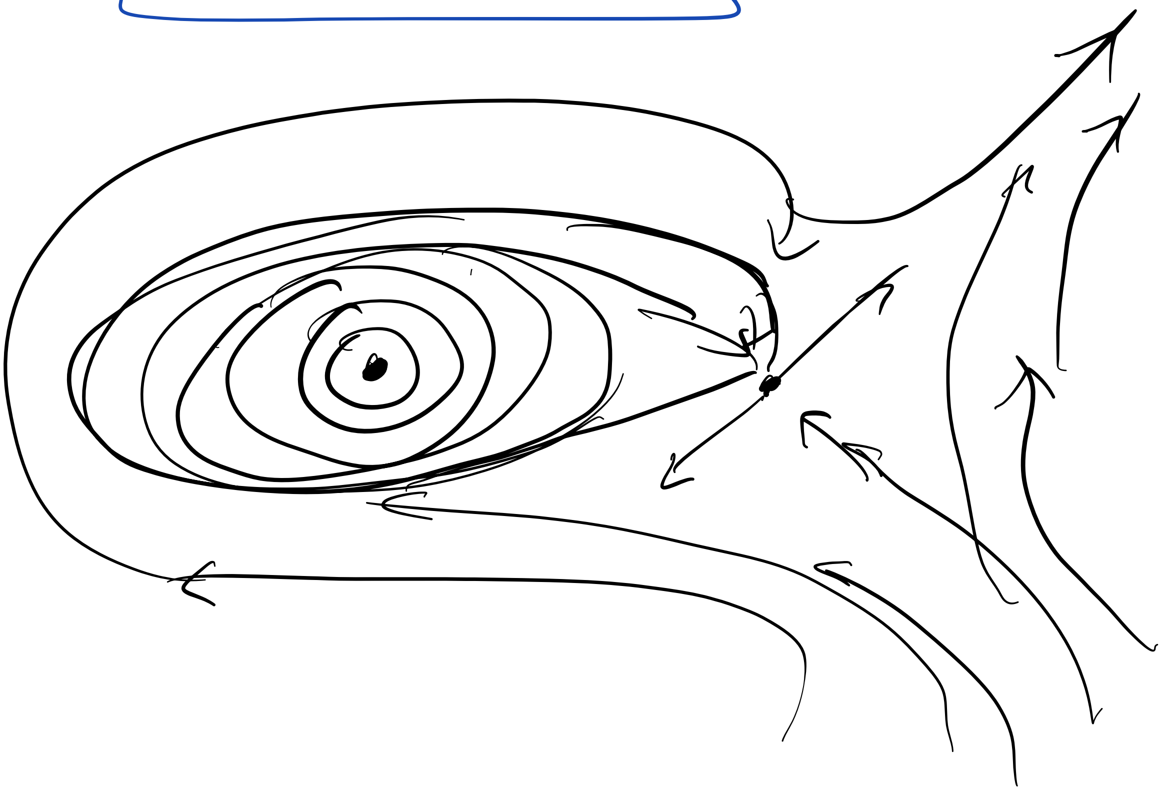
$$x' = y = 0 \rightarrow y = 0$$

Ex:

$$y' = -x + x^2 = 0 \rightarrow x: 0, 1$$

Find the critical points.

$\boxed{(0,0) \qquad (1,0)}$



Ex.

$$x' = x(3 - x - 2y) \qquad (1)$$

$$y' = y(4 - 2y - 4x) \qquad (2)$$

we can start with $y$

$y=0$

$4-2y-4x=0$

$4-4x=2y$

$2-2x=y$

$\underline{y=2-2x}$

$y=0$

sub (1)

$x(3-x-0)=0$

$x(3-x)=0$

$\underline{x=0}$ $\quad$ $\underline{x=3}$

$\underline{y=0}$

$(0,0)$ $\quad$ $(3,0)$

$y = 2 - 2x$

sub (1)

$x(3 - x - 2(2 - 2x)) = 0$

$x(3 - x - 4 + 4x) = 0$

$x(-1 + 3x) = 0$

$x(3x - 1) = 0$

$x = 0, \quad x = \frac{1}{3}$

$y = 2 - 2x$ ?

$2 - 2(0) = 2$

$\boxed{(0, 2)}$

$2 - 2\left(\frac{1}{3}\right) = \boxed{\left(\frac{1}{3}, \frac{4}{3}\right)}$

CP:

$\boxed{(0,0)}$ $\boxed{(3,0)}$

$\boxed{0, 2}$ $\boxed{\left(\frac{1}{3}, \frac{4}{3}\right)}$

Ex.

$x' = 1 + 5y$

$y' = 1 - 6x^2$

Find all CP.

$y = -\frac{1}{5}$

$x = \pm \frac{1}{\sqrt{6}}$

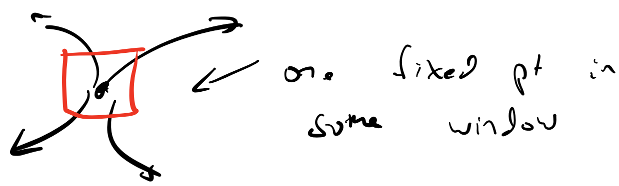
$\left(\frac{1}{\sqrt{6}}, -\frac{1}{5}\right)$

$\left(-\frac{1}{\sqrt{6}}, -\frac{1}{5}\right)$

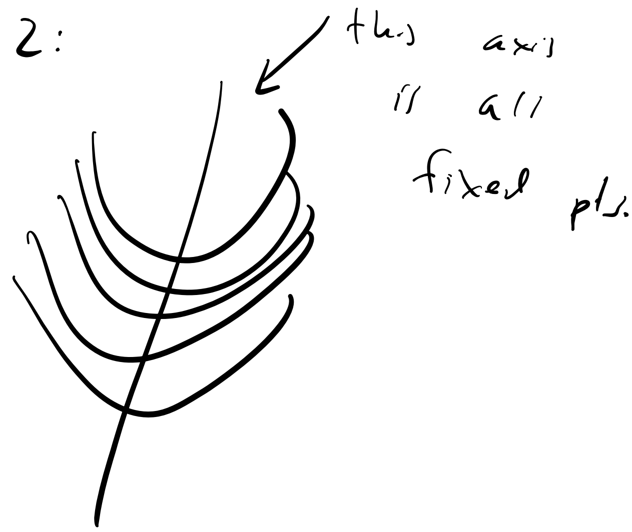
A system $Ax + g$ is almost linear around a point $d$ if

1) the Jacobian of $A$ is invertible, and

2) $d$ is locally unique




Violating 2:




fixed pt = critical pt = eq. pt.

Ex. 7.2

Write the approximating linear system near $(0,2)$:

$$x' = x(3 - x^2 - 2y)$$
$$y' = y(4 - 8y - 4x)$$

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$$F \quad x' = 3x - x^3 - 2xy$$
$$G \quad y' = 4y - 8y^2 - 4xy$$

$$J = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} = \begin{pmatrix} 3 - 3x^2 - 2y & -2x \\ -4y & 4 - 16y - 4x \end{pmatrix}$$

$\partial f$ $\qquad\qquad$ $\partial f$

$\overline{\partial x}$ $\overline{\partial y}$

treat x as a variable

treat y as a variable

else is constant

A :)

$$\begin{pmatrix} 3-3x^2-2y & -2x \\ -4y & 4-16y-4x \end{pmatrix} \xrightarrow{(0,2)} \begin{pmatrix} -1 & 0 \\ -8 & -28 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -8 & -28 \end{pmatrix} \begin{pmatrix} x \\ y-2 \end{pmatrix}}$$

Follow-up:

is (0,2)

a) stable

b) unstable

c) saddle pt

d) other