Office Hours

1-3 PM Thurs.

Shifted Systems

Monday, 6 PM Review Sessions

S.S. $\vec{r}$ = vector

1) $x' = A(x - \vec{r})$

$x = C_1 e^{\lambda_1 t} v_1 + C_2 e^{\lambda_2 t} v_2 + \vec{r}$

2) $x' = Ax + b$

$$x = C_1 e^{\lambda_1 t} v_1 + C_2 e^{\lambda_2 t} v_2 + r$$

s.t. $Ar + b = 0$

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e.x.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 - 3 \\ x_2 + 1 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - v$$

$$\lambda^2 - 4\lambda + 3$$

$$(\lambda - 3)(\lambda - 1)$$

$$\lambda_1 = 1 \qquad \lambda_2 = 3$$

$\lambda_1 = 1$:

$$\begin{bmatrix} 2-1 & 1 \\ 1 & 2-1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1 + v_2 = 0$$

$\eta_1 = -\eta_2$ $\quad \begin{bmatrix} -\eta_2 \\ \eta_2 \end{bmatrix}$

$v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\leftarrow \eta_2 = 1$

$\lambda_2 = 3$

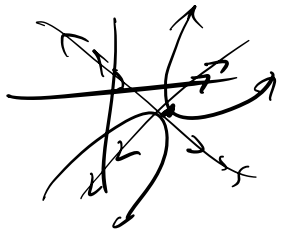
$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-1\eta_1 + 1\eta_2 = 0$

$\eta_1 = \eta_2$ $\quad \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$

$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \eta_2 = 1$

G.S. of shifted:



$$y = C_1 e^{t}\begin{bmatrix}-1\\1\end{bmatrix} + C_2 e^{3t}\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}3\\-1\end{bmatrix}$$

Ex 2. $x' = Ax + b$

$$x' = \begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}0\\-1\end{bmatrix}$$

Set 0

$$\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$$2x_1 + x_2 = 0$$

$$x_1 + 2x_2 = 1$$

$$x_2 = -2x_1$$

$$x_1 + 2(-2x_1) = 1$$

$$x_1 - 4x_1 = 1$$

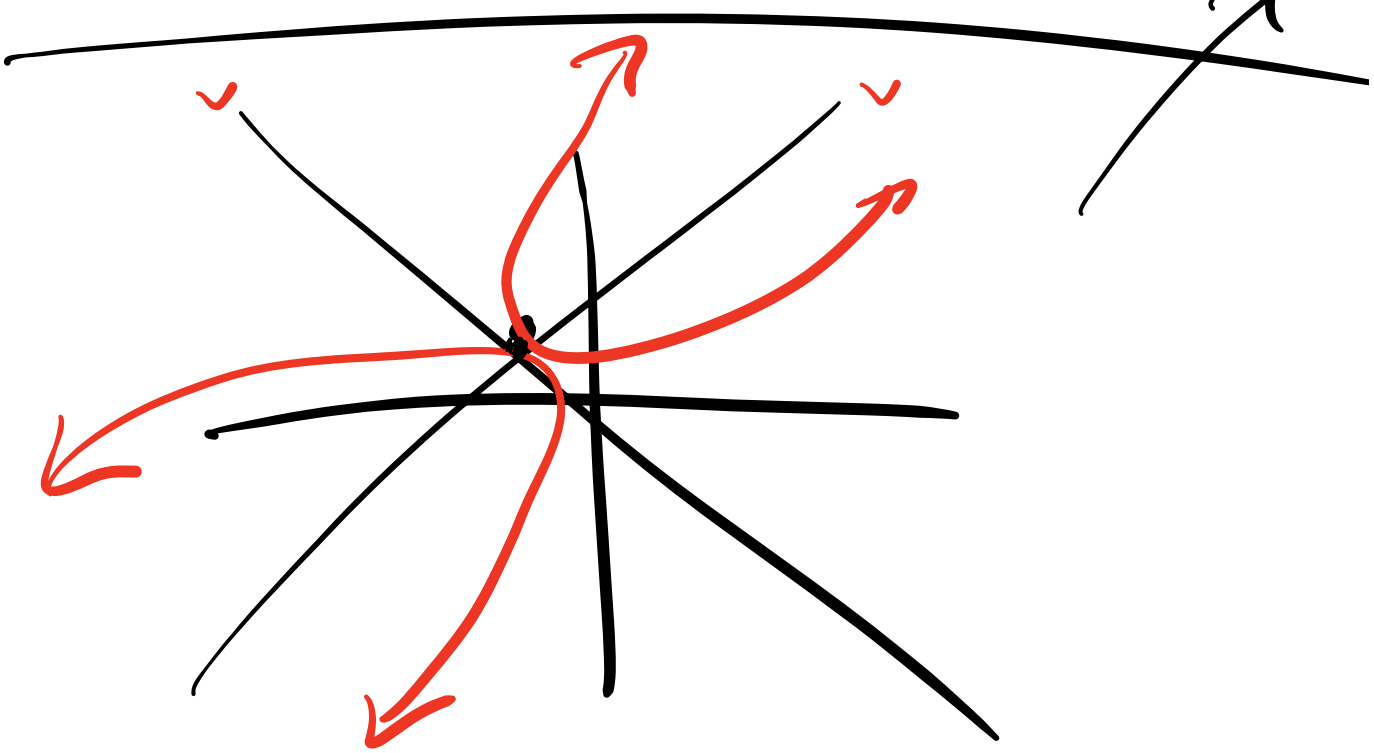
$$-3x_1 = 1$$

$$x_1 = -\frac{1}{3}$$

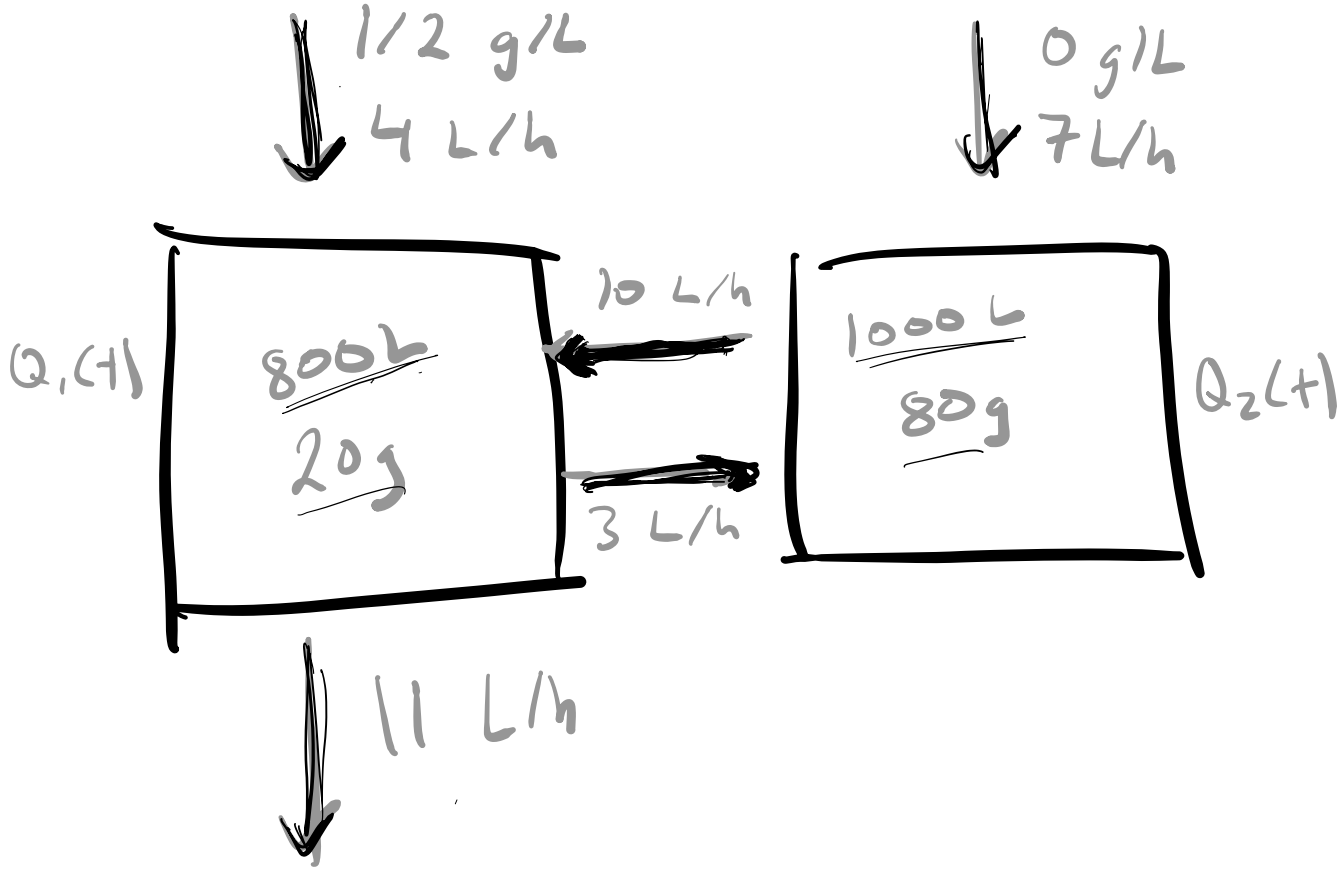
$$x_2 = \frac{2}{3}$$

$y =$

$$C_1 e^{t}\begin{pmatrix} -1 \\ 1 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{bmatrix} -1/3 \\ 2/3 \end{bmatrix}$$



2 Salt Tanks



$$\frac{dQ}{dt} = c_i(t)\, r_i(t) - \left\{ \frac{Q(t)}{V(t)} \right\} r_o(t)$$

in ↗ ↑ out

1) Salt entering externally

2) Salt entering from other tank

3) salt leaving

$$\underset{cr}{1} + \underset{cr}{2} - \underset{cr}{3}$$

Q_1:

In

1) $4\left(\frac{1}{2}\right)$

2) $10\left(\frac{Q_2}{1000 + 10t - 10t}\right)$

out 13) $14\left(\frac{Q_1}{800 + 14t - 14t}\right)$

$$\frac{dQ_1}{dt} = 1 + 2 - 3$$

$$= 4\left(\frac{1}{2}\right) + 10\left(\frac{Q_2}{1000}\right) - 14\left(\frac{Q_1}{800}\right)$$

$$\frac{dQ_1}{dt} = 2 + \frac{Q_2}{100} - \frac{7Q_1}{400}$$

$$Q_1(0) = 20$$

$$\frac{dQ_2}{dt} =$$

1) Salt entering externally
2) Salt entering from other tank
3) salt leaving

1) 0

2) $3\left(\frac{Q_1}{800 + 14t - 4t}\right)$

3) $10\left(\frac{Q_2}{\quad}\right)$

$$\frac{dQ_2}{dt} = 1 + 2 - 3$$

$$= \frac{3Q_1}{800} - \frac{Q_2}{100}$$

$$Q_2(0) = 80$$

$$\frac{dQ_1}{dt} = 2 + \frac{Q_2}{100} - \frac{7Q_1}{400}$$

$$\frac{dQ_2}{dt} = \frac{3Q_1}{800} - \frac{Q_2}{100}$$

$$Q_1(0) = 20$$

$$Q_2(0) = 80$$

$$Q' = \begin{bmatrix} -\frac{7}{400} & \frac{1}{100} \\ \frac{3}{800} & -\frac{1}{100} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$Q(0) = \begin{bmatrix} 20 \\ 80 \end{bmatrix}$$