

Feb 28

Quiz 3 Today

Method of Undetermined Coefficients

Second-order diff. eqn with constant
coefficients: (non-homogeneous)

$$y'' + p(t)y' + q(t)y = g(t)$$

$$y'' + p(t)y' + q(t)y = 0$$

Complementary Solution

Particular Solution = ?

General Solution = C.S. + P.S.

To find P.S:

- Look at RHS $g(t)$
- Make a guess abt P.S. based on $g(t)$.
 - Leave the coefficients undetermined
- Try solution
 - It works

Ex: $y'' - 4y' - 12y = 3e^{5t}$

C.S. $y'' - 4y' - 12y = 0$

$$r^2 - 4r - 12 = 0$$

$$(r - 6)(r + 2)$$

$$r = 6, -2$$

$$C.S. = \underline{C_1 e^{6t} + C_2 e^{-2t}}$$

$$P.S. = g(t) = 3e^{5t}$$

$g(t)$	Guess
$a e^{\beta t}$	$A e^{\beta t}$
$a \cos(\beta t)$	$A \cos(\beta t) + B \sin(\beta t)$
$a \sin(\beta t)$	$A \sin(\beta t) + B \cos(\beta t)$
t	$(A + B)t$
t^2	$(A t^2 + B t + C)$

$$y_p(t) = A e^{5t}$$

$$y_p'(t) = 5A e^{5t}$$

$$y_p''(t) = 25A e^{5t}$$

$$y'' - 4y' - 12y = 3e^{5t}$$

$$25Ae^{5t} - 4(5Ae^{5t}) - 12Ae^{5t} = 3e^{5t}$$

$$-7Ae^{5t} = 3e^{5t}$$

$$A = -\frac{3}{7}$$

$$y_p(t) = -\frac{3}{7}e^{5t}$$

$$\text{G.S.} = C_1 e^{6t} + C_2 e^{-2t} - \frac{3}{7} e^{5t}$$