Plan Today:

- Talk about quiz
  - Send review/practice problems
- Extra Office Hours 7-8PM
  Zoom
- Office Hours tomorrow
  (Tues) Math Lab
  Clough 280 1-2 PM.
- Phase Planes
  Complex Eigenvalues
  Repeated Eigenvalues

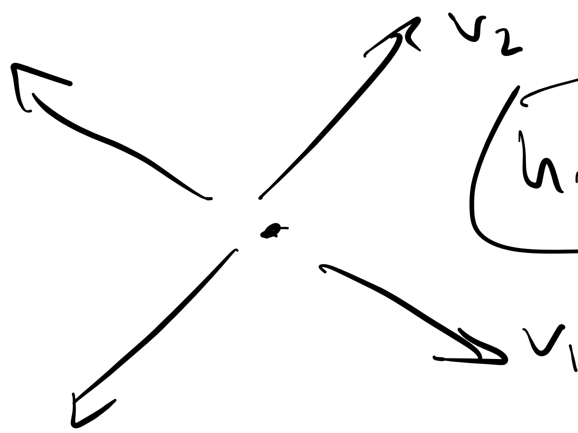
Phase Planes

Quiz 2

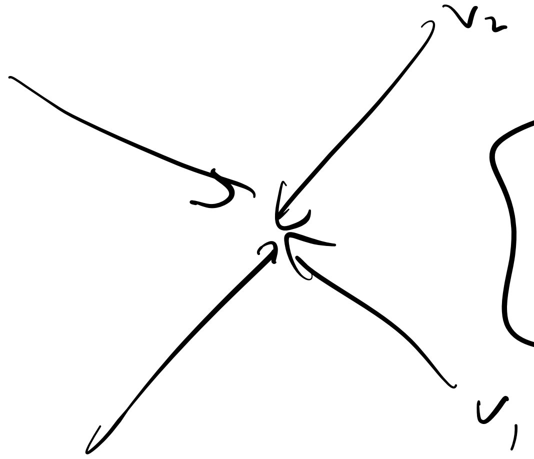
1) SoE

Plot a phase portrait

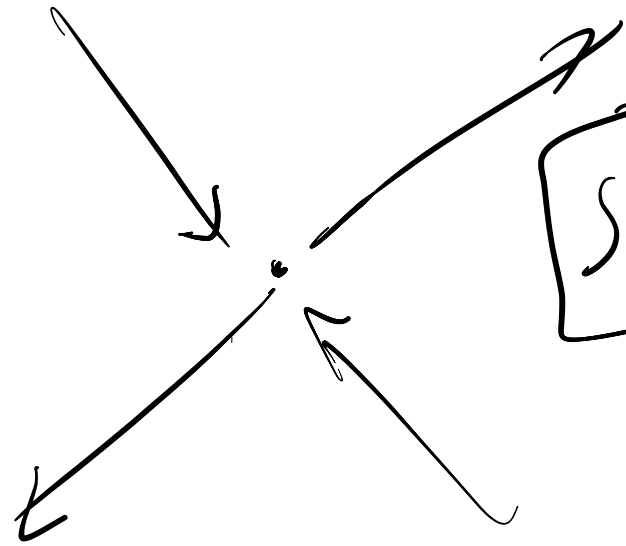
2 real, unique eigenvalues




Unstable Node




Stable Node



Saddle Pt.

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2) Solve IVP.

Sol:

3) interval of existence,
classifying order,

transform ODE into system.

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$$X' = \begin{pmatrix} 3 & -13 \\ 5 & 1 \end{pmatrix} X,$$

$$X(0) = \begin{pmatrix} 3 \\ -10 \end{pmatrix}$$

$$\begin{vmatrix} 3-\lambda & -13 \\ 5 & 1-\lambda \end{vmatrix}$$

$$(3-\lambda)(1-\lambda) - (-13)(5)$$

$$3 - \lambda - 3\lambda + \lambda^2 + 65$$

$$\lambda^2 - 4\lambda + 68 = 0$$

$-256$

$$\lambda = \frac{4 \pm \sqrt{4^2 - 4(1)(68)}}{2}$$

$$= 2 \pm 8i$$

$\underline{2 + 8i}$

$$\begin{bmatrix} 3-\lambda & -13 \\ 5 & 1-\lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-(2+8i) & -13 \\ 5 & 1-(2+8i) \end{bmatrix}$$

$$= \begin{bmatrix} 1-8i & -13 \\ 5 & -1-8i \end{bmatrix}$$

$$\begin{bmatrix} 1-8i & -13 \\ 5 & -1-8i \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5\eta_1 + (-1-8i)\eta_2 = 0$$

$$5\eta_1 = (1+8i)\eta_2$$

$$\eta_1 = \frac{(1+8i)}{5}\eta_2$$

$$v_1 = \begin{pmatrix} \frac{(1+8i)}{5}\eta_2 \\ \eta_2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+8i \\ 5 \end{pmatrix} \Rightarrow \cancel{\begin{pmatrix} 1-8i \\ 5 \end{pmatrix}}$$

$$X_1 = e^{\lambda t}(v)$$

$$= e^{(2+8i)t}\begin{pmatrix} 1+8i \\ 5 \end{pmatrix}$$

$$= e^{2t} e^{(8i)t} \begin{pmatrix} 1+8i \\ 5 \end{pmatrix}$$

root $= d \pm \mu i$

$$e^{dt} e^{(i\mu)t} =$$

$$e^{dt}\left(\cos(\mu t) + i\sin(\mu t)\right)$$

$$= e^{2t}\left(\cos(8t) + i\sin(8t)\right)\begin{pmatrix} 1+8i \\ 5 \end{pmatrix}$$

$$R \quad e^{2t}\begin{pmatrix} \cos(8t) - 8\sin(8t) \\ 5\cos(8t) \end{pmatrix} +$$

$$I \quad e^{2t}\begin{pmatrix} 8i\cos(8t) + i\sin(8t) \\ 5i\sin(8t) \end{pmatrix}$$

$$= e^{2t}\begin{pmatrix} \cos(8t) - 8\sin(8t) \\ 5\cos(8t) \end{pmatrix} +$$

$$i e^{2t}\begin{pmatrix} 8\cos(8t) + \sin(8t) \\ 5\sin(8t) \end{pmatrix}$$

$$= C_1 e^{2t}\begin{pmatrix} \cos(8t) - 8\sin(8t) \\ 5\cos(8t) \end{pmatrix} +$$

$$C_2 e^{2t}\begin{pmatrix} 8\cos(8t) + \sin(8t) \\ 5\sin(8t) \end{pmatrix}$$

$$x(0) = \begin{pmatrix} 3 \\ -10 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ -10 \end{pmatrix} = C_1\begin{pmatrix} \overset{1}{\cancel{\cos(8t)}} \overset{0}{\cancel{- 8\sin(8t)}} \\ \underset{5}{\cancel{5\cos(8t)}} \end{pmatrix}$$

$$+ C_2\begin{pmatrix} 8\,\cancel{\cos(8t)} + \cancel{\sin(8t)} \\ \cancel{5\sin(8t)} \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ -10 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 5 \end{pmatrix} + c_2 \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$3 = c_1 + 8c_2$$

$$-10 = 5c_1$$

$$\boxed{c_1 = -2}$$

$$3 = -2 + 8c_2$$

$$5 = 8c_2$$

$$\boxed{c_2 = \frac{5}{8}}$$

IVP Solution:

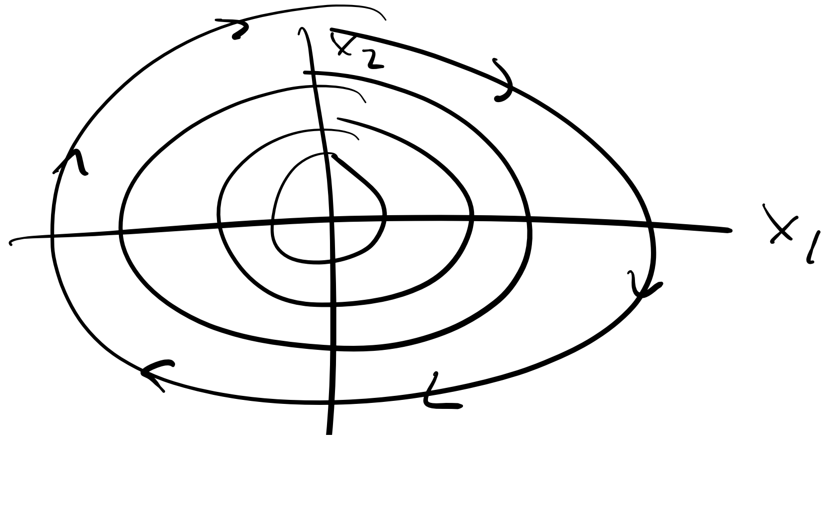
$$\boxed{x = -2e^{2t}\begin{pmatrix} \cos(8t) - 8\sin(8t) \\ 5\cos(8t) \end{pmatrix} + \frac{5}{8}e^{2t}\begin{pmatrix} 8\cos(8t) + \sin(8t) \\ 5\sin(8t) \end{pmatrix}}$$

Complex Plots

root: $\underset{r}{\alpha} \pm \underset{i}{\mu i}$

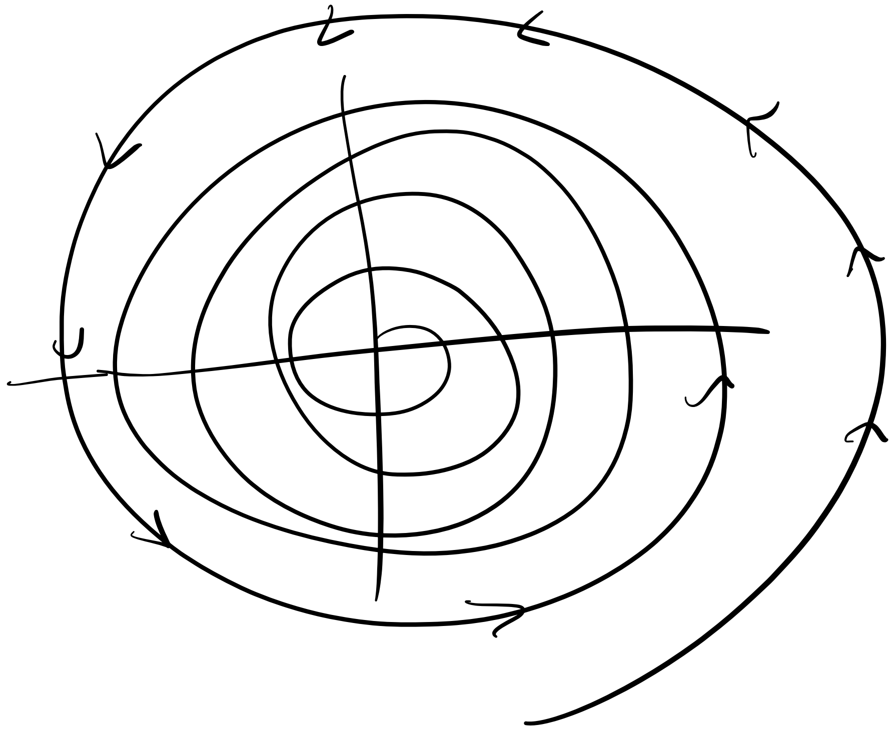
No real part:

$\boxed{\text{Center}}$ – stab



Real part:

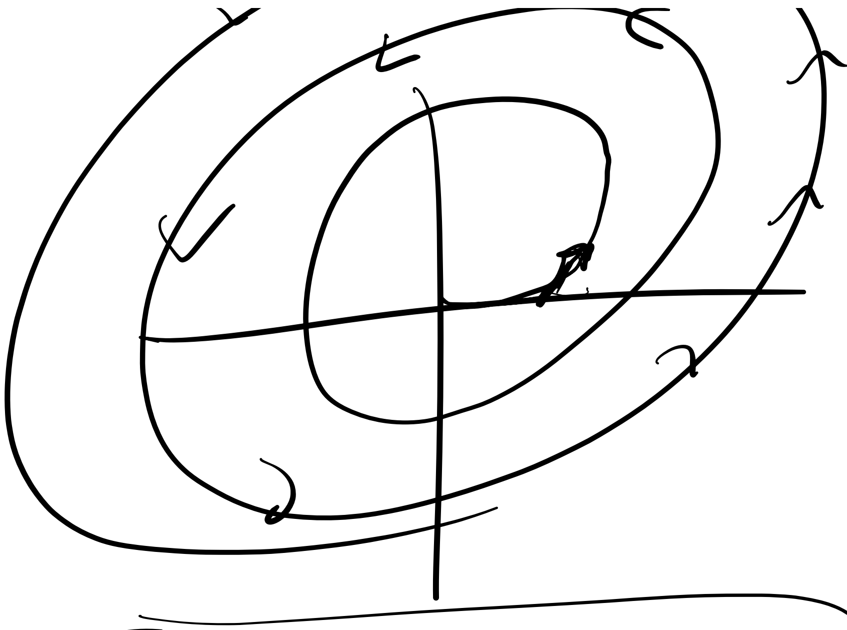
$\boxed{\text{Spiral}}$



Numerically test

direction:

$$\begin{pmatrix} 3 & -13 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$



Unstable spiral

Wksht 3.4

Q2:

Find G.S.

Plot phase plane:

$$X' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} X$$

## Repeated Eigenvalues:

$$x' = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} x$$

$$\left| \begin{pmatrix} 7-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} \right| = 0$$

$$(7-\lambda)(3-\lambda) - (1)(-4) = 0$$

$$\lambda^2 - 10\lambda + 25 = 0$$

$$(\lambda_1 - 5)(\lambda_2 - 5) = 0$$

$\lambda_{1,2} = 5$

$$\begin{pmatrix} 7-5 & 1 \\ -4 & 3-5 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$2\eta_1 + \eta_2 = 0$

$\eta_2 = -2\eta_1$

$$v_1 = \begin{pmatrix} \eta_1 \\ -2\eta_1 \end{pmatrix}$$

Choose $\eta_1 = 1$

$$v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$2\eta_1 + 1\eta_2 = 1$

$\eta_2 = 1 - 2\eta_1$

$$\eta_1 = 0 \qquad \begin{pmatrix} \eta_1 \\ 1-2\eta_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

We can choose 0 now.

$$x(t) = C_1 e^{5t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2\left(t e^{5t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + e^{5t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$$

$$\boxed{x(t) = C_1 e^{5t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{5t}\left(t\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)}$$

$$\begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$$

$$\boxed{\text{improper node}}$$