

Office Hours Tues. 1-2 PM
Clough 280 (Math Lab)
or by appointment

1.3, 2.1, 2.2

Sec 1.3

- Classification of DE's
- Standard Form

Classification of DE's

$$\text{ODE's} = y(t)$$

$$\text{PDE's} = y(t, x_1, x_2, x_3, \dots)$$



$$\frac{\partial u(x,t)}{\partial t} = D \frac{\partial^2 u}{\partial x^2}$$

$$\hookrightarrow \frac{du(t)}{dt} = ku$$

order:

largest degree derivative
in equation.

$$\hookrightarrow y'' + y = k(x)$$

2nd - order DE

$$y' = y$$

1st - order DE

Linear DE

only linear terms of unknown function and the derivatives

$$\underline{y' = y}$$

$$\frac{y = e^t}{\text{General Solution}}$$

$$y^{(4)} + y = y^{(1)}$$

$$A(x)y = y''$$

~~$$y' = y$$~~

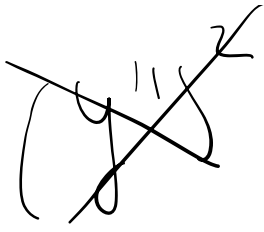
~~$$(y)''$$~~

Nonlinear DE

contains non-linear term of unknown functions.

$$y' y = 22$$

G.S. dne usually



Linear:

$$a_n(x) \frac{d^n y}{dx^n} + \dots + a_1(x) \frac{dy}{dx} +$$

$$a_0(x)y = b(x)$$

Standard Form:

$$y^n + \dots + A(x)y'' + B(x)y' + C(x)y = D(x)$$

$$\exists y'' = y$$



S.F

$$3y'' - y = 0$$

$$y'' - \frac{1}{3}y = 0 \quad \text{S.F.}$$

S.F.

1) coeff. of highest degree term is 1

2) all unknown functions on LHS.

Ex. 1.3

$$y \frac{dy}{dt} = t$$

Linear? No

2.1 Separation of Variables

"variable separable"

You can use if:

$$\frac{dy}{dx} = f(y)g(x)$$

solve.

$$dy = f(y)g(x) dx$$

$$\frac{dy}{f(y)} = g(x) dx$$

$$\int \frac{dy}{f(y)} = \int g(x) dx$$

$$\int \frac{1}{f(y)} dy = \int g(x) dx$$

$$F(y) = G(x) + C$$

$$y = ?$$

Exo

Solve the DE using

SoV.

$$\frac{dy(x)}{dx} = \frac{e^x}{y}$$

$$y \, dy = e^x \, dx$$

$$\int y \, dy = \int e^x \, dx$$

$$\frac{1}{2} y^2 = e^x + C$$

$$y^2 = 2e^x + C$$

$$y = \pm \sqrt{2e^x + C}$$

Ex.

WS 2.2

$$\frac{dy}{dx} = x e^{x+y}$$

Solve using SoV.

$$\frac{dy}{dx} = x e^x e^y$$

$$\frac{dy}{e^y} = x e^x dx$$

$$e^{-y} dy = x e^x dx$$

$$\int e^{-y} dy = \int x e^x dx$$

$$-e^{-y} = \int x e^x dx \quad \text{I.B.P.}$$

$$f = x \quad g' = e^x$$

$$f' = 1 \quad g = e^x$$

$$x e^x - \int e^x$$

$$-e^{-y} = x e^x - e^x + C$$

2.2 Integrating Factor

Consider 1st-order DE. in S.F.

$$y' + P(x)y = H(x)$$

Assume term $\mu(x)$ exists

↓

$$\mu(x)y' + P(x)y\mu(x) = \mu(x)H(x)$$

↓

This will make integrating easy.

$$\mu(x) = e^{\int P(x) dx}$$

Ex. W.S. 2.2

$$ty' + 2y = \sin(t)$$

Solve the DE using I.F.

1) Put in S.F.

$$y' + \frac{2}{t}y = \frac{\sin(t)}{t}$$

$$2) \mu(t) = e^{\int \frac{2}{t} dt} = e^{2 \ln |t|}$$

$$n \log_b m = \log_b (m^n)$$

$$e^{\overbrace{2 \ln |t|}^{\rightarrow}} = e^{\ln |t^2|}$$

$$\downarrow e^{\ln(t^2)}$$

$$\mu(t) = t^2$$

$$3) t^2 y' + t \frac{2}{t} y = \frac{\sin(t)}{t} t^2$$

$$t^2 y' + 2ty = t \sin(t)$$

$$4) \int (t^2 y' + 2ty)^{dy} = \int t \sin(t) dt$$

$$\Rightarrow \int (t^2 y)' = \int t \sin(t) dt$$

$$\Rightarrow t^2 y = \downarrow$$

$$f = t$$

$$f' = 1$$

$$g' = \sin(t)$$

$$g = -\cos(t)$$

$$\begin{aligned} & \downarrow \\ & -t \cos(t) - \int -\cos(t) \\ t^2 y & = -t \cos(t) + \sin(t) + C \end{aligned}$$

$$y = \frac{-t \cos(t) + \sin(t) + C}{t^2}$$

$$y = \frac{-\cos(t)}{t} + \frac{\sin(t)}{t^2} + \frac{C}{t^2}$$

Quiz | Next week

Last semester: 3 questions

I will send an email
at least w/ similar problems.
