Office Hours Tues. 1-2PM

Clough 280 (Math Lab)

or by appointment

1.3, 2.1, 2.2

Sec 1.3

- Classification of DE's
- Standard Form

Classification of DE's

ODE's = $y(t)$

PDE's : $y(t, x_1, x_2, x_3 \dots)$

$$\frac{\partial}{\partial t} u(x,t) = D \frac{\partial u^2}{\partial x^2}$$

i) $$\frac{du(t)}{dt} = ku$$

order:

largest degree derivative in equation.

$$y'' + y = k(x)$$

$\hookrightarrow$ 2^{nd}-order DE

$$y' = y$$

1^{st}-order DE

Linear DE

only linear terms of unknown function and the derivatives

$$y' = y$$

$y = e^t$
General Solution

$$y' + y = y''$$

$$A(x)\, y = y''$$

$$\cancel{y' y}$$

$$\cancel{(y)^2}$$

Nonlinear DE

Contains non-linear term of unknown functions.

$$y' y = 22$$

G.S. dne
usually

$\cancel{(y'')^2}$

Linear:

$$a_n(x)\frac{d^n y}{dx^n} + \dots + a_1(x)\frac{du(x)}{x} +$$

$$a_0(x)u(x) = b(x)$$

Standard Form:

$$y^n + \dots A(x)y'' + B(x)y'$$

$$+ C(x)y = D(x)$$

$$3y'' = y$$

$$\Big\downarrow \quad \text{S.F}$$

$$3y'' - y = 0$$

$$y'' - \frac{1}{3}y = 0 \qquad \text{S.F.}$$

S.F.

1) coeff. of highest degree term is 1

2) all unknown functions on LHS.

Ex. 1.3

$$y\frac{dy}{dt} = t$$

Linear? $\boxed{\text{No}}$

2.1 Separation of Variables

"variable separable"

You can use if:

$$\frac{dy}{dx} = f(y)\,g(x)$$

solve.

$$dy = f(y)\,g(x)\,dx$$

$$\frac{dy}{f(y)} = g(x)\,dx$$

$$\int \frac{dy}{f(y)} = \int g(x)\, dx$$

$$\int \frac{1}{f(y)}\, dy = \int g(x)\, dx$$

$$F(y) = G(x) + C$$

$$y = ?$$

Ex.

Solve the DE using SoV.

$$\frac{dy(x)}{dx} = \frac{e^x}{y}$$

$$y\,dy = e^x\,dx$$

$$\int y\,dy = \int e^x\,dx$$

$$\frac{1}{2}y^2 = e^x + C$$

$$y^2 = 2e^x + C$$

$$\boxed{y = \pm\sqrt{2e^x + C}}$$

Ex.

ws 2.2

$$\frac{dy}{dx} = xe^{x+y}$$

Solve using SoV.

$$\frac{dy}{dx} = xe^{x}e^{y}$$

$$\frac{dy}{e^{y}} = xe^{x}\,dx$$

$$e^{-y}\,dy = xe^{x}\,dx$$

$$\int e^{-y}\,dy = \int xe^{x}\,dx$$

$$-e^{-y} =$$

I.B.P.

$f = x \quad g' = e^{x}$

$f' = 1 \quad g = e^{x}$

$$xe^{x} - \int e^{x}$$

$$\boxed{-e^{-y} = xe^{x} - e^{x} + C}$$

2.2 Integrating Factor

Consider 1^{st}-order D.E. in S.F

$$y' + P(x)y = H(x)$$

Assume term $\mu(x)$ exists

$\downarrow$

$$\mu(x)y' + P(x)y\mu(x) = \mu(x)H(x)$$

$\downarrow$

This will make integrating easy.

$$\mu(x) = e^{\int P(x)\,dx}$$

Ex. W.S 2.2

$$ty' + 2y = \sin(t)$$

Solve the D.E. using I.F.

1) Put in S.F.

$$y' + \frac{2}{t}y = \frac{\sin(t)}{t}$$

2) $\mu(t) = e^{\int \frac{2}{t}\,dt} = e^{2\ln|t|}$

$n \log_b m = \log_b(m^n)$

$$e^{2\ln|t|} = e^{\ln|t^2|}$$

$$\downarrow \cancel{e^{\ln}}(t^2)$$

$$\mu(t) = t^2$$

3) $t^2 y' + t^2 \frac{2}{t} y = \frac{\sin(t)}{t} t^2$

$$t^2 y' + 2ty = t\sin(t)$$

4) $$\int (t^2 y' + 2ty) \overset{dy}{=} \int t\sin(t)\,dt$$

$$\Rightarrow \int (t^2 y)' = \int t\sin(t)\,dt$$

$$\Rightarrow t^2 y =$$

$f = t \qquad g' = \sin(t)$

$f' = 1 \qquad g = -\cos(t)$

$$-t\cos(t) - \int -\cos(t)$$

$$t^2 y = -t\cos(t) + \sin(t) + C$$

$$y = \frac{-t\cos(t) + \sin(t) + C}{t^2}$$

$$\boxed{y = \frac{-\cos(t)}{t} + \frac{\sin(t)}{t^2} + \frac{C}{t^2}}$$

Quiz | next week

Last semester: 3 questions

I will send an email

at least w/ similar problems.