Today: Quiz Review

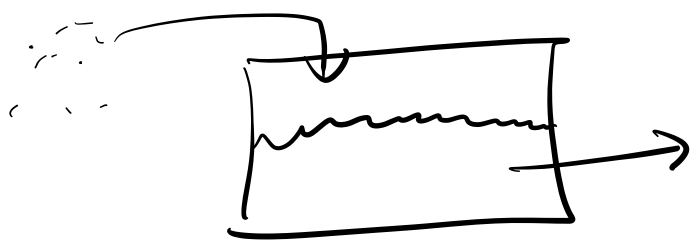
2.3, 2.4

Modelling

Existence + Uniqueness

Diff Eq. → populations

rate of changes



Salt Tank: Modelling

$Q(t)$ = amount of salt

Rate of change of $Q(t)$ $= \dfrac{dQ}{dt} = Q'(t)$

$$\frac{dQ}{dt} = \underbrace{c_i(t)r_i(t)}_{\text{salt entering}} - \underbrace{\frac{r_o(t)\,Q(t)}{V(t)}}_{\text{salt exiting}}$$

Ex.

A tank initially contains 200 gallons of fluid where 20 lbs of salt are dissolved. Brine containing 3 lbs salt/gallon enters at a rate of 5 gallon/min. Exits 5 gallons/minute. $Q(t)$ is salt at time t.

Write the DE and initial condition.

$$\frac{dQ}{dt} = \underbrace{c_i(t)r_i(t)}_{3(5)} - \frac{Q(t)}{V(t)}\,r_o(t)$$

$$15 - \frac{5\,Q(t)}{200}$$

$$\boxed{\frac{dQ}{dt} = 15 - \frac{Q(t)}{40}, \quad Q(0) = 20}$$

Ex.

10,000 bunnies in our city. Bunnies reproduce at an annual growth rate of 12%. Every month, 50 bunnies die. Let p(t) be the current number of bunnies at time t. Let r be the growth rate for the compounding

period 1. Write the DE
and initial conditions.

$$p(0) = 10{,}000$$

$$\frac{dp}{dt} \quad .12p - 600$$

$$\frac{dp}{dt} = \frac{\text{bunnies}}{\text{accumulated}} - \frac{\text{bunnies}}{\text{lost}}$$

2.4

Ex. Un. Thm.

Consider IVP:

$$\begin{cases} y' + p(t)y = g(t), \\ y(t_0) = y_0 ; \end{cases}$$

If $p(t)$ and $g(t)$ are continuous functions on interval $\alpha < t < \beta$ and the interval contains t_0, then there is a unique solution to IVP on that interval.

$$\begin{cases} y' + \frac{1}{x} y = 4 \\ y(0) = 1 \end{cases}$$

$y' + p(x) y = f(x)$

$$y' + \frac{1}{x} y = 4$$

$$y(\boxed{1}) = 2$$

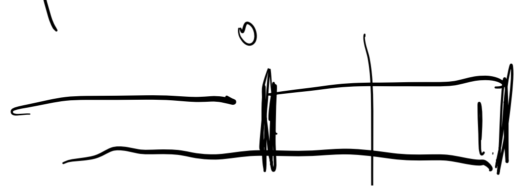
$$\boxed{(0, \infty)}$$

$(-\infty, 0) \quad (0, \infty)$

$\frac{1}{x} \rightarrow$

$(-\infty, \infty)$

$4 \rightarrow$

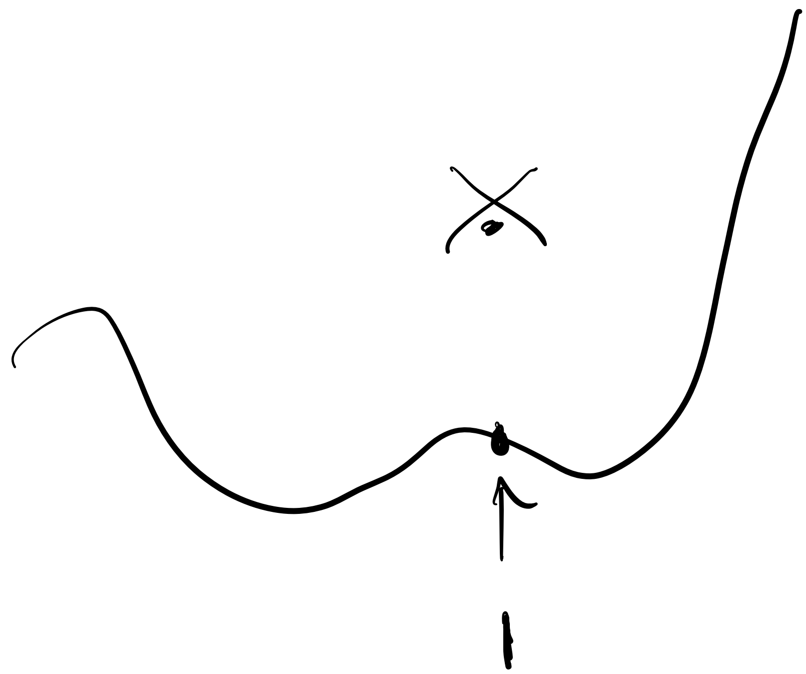


1

$t_0 = 1$

P.S.

$y =$



Ex. Determine the interval of existence?

$$\left\{ ty'' + 3y = t \right. \qquad \left[\begin{array}{l} y(1) = 1 \\ y'(1) = 2 \end{array} \right.$$

$$y'' + P(t)\,y' + q(t)\,y = g(t)$$

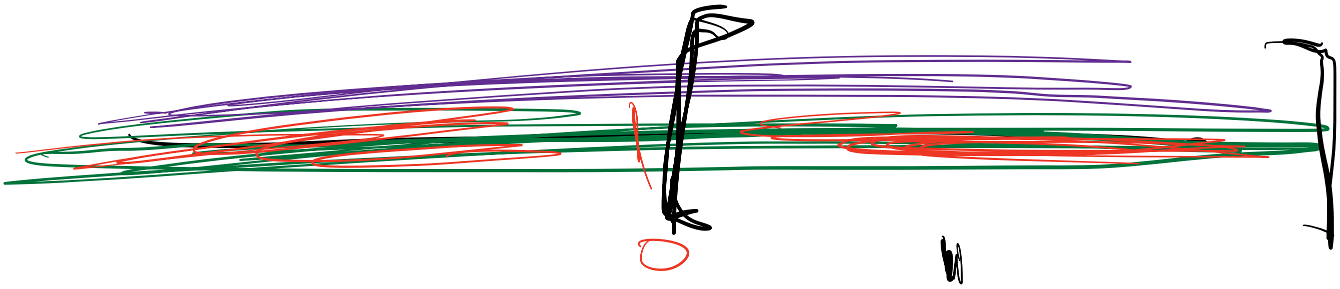
$$y'' + \frac{3}{t}y = 1$$

$$y'' + [0]\,y' + \left[\frac{3}{t}\right]y = [1]$$

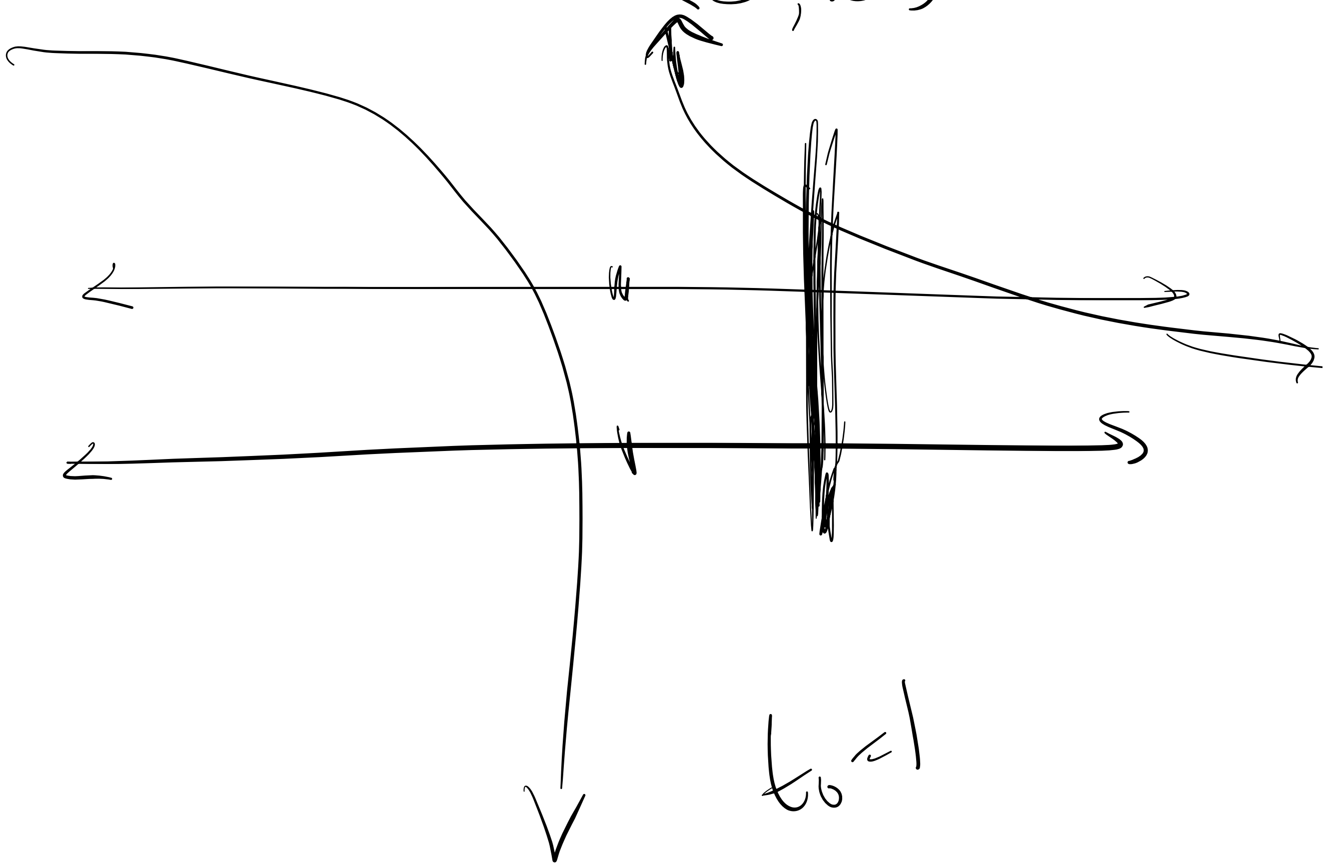
$0 \longrightarrow (-\infty, \infty)$

$\frac{3}{t} = (-\infty, 0)\ (0, \infty)$

$1 = (-\infty, \infty)$



$(-\infty, 0)$ $(0, \infty)$



$t_0 = 1$

$\boxed{(0, \infty)}$