2^{nd} - order DE

Systems of DE

Systems? y_1 y_2 = unknowns

$$\begin{cases} y_1' = \xi y_1 + \xi y_2 \\ y_2' = \xi y_1 + \xi y_2 \end{cases} = \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} \xi & \xi \\ \xi & \xi \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Solve for y_1, y_2

Homogeneity:

Homogeneous	Non-homogeneous
SF	
RHS = 0	RHS ≠ 0
$y' - y = 0$	$y' - y = 2$
~~$y' - y + 2 = 0$~~	

Autonomy

Autonomous

$y' = f(y)$

N-A

$y' = yt$

$y' = y + t$

$y' = f(y, t)$

Linear Homogenous DE's:

$y' = y$

$y' - y = 0$

$y_1 = e^t$

$y_2 = 3e^t$

y_1, y_2 are solutions

$c_1 y_1 + c_2 y_2$ is also a solution

SOE:

Predator - prey

Worksheet 3.2

Q1

$$\begin{cases} x' = -x + ty \\ y' = t - y \end{cases}$$

Write in matrix form:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} -1 & t \\ t & -1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

NA, H

2) wksht 3.2

2nd order ODE → S.E

$$u'' - 2u' + u = \sin(t)$$

Sol.

let $x_1 = u$

$x_2 = u'$

$$x_1' = u' = x_2$$

$$x_2' = u'' = 2u' - u + \sin(t)$$

$$u'' = 2x_2 - x_1 + \sin(t)$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \sin(t) \end{pmatrix}$$

Midterm L.S.

SoE:

$$\left(\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \vec{x}' \right) \qquad \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \vec{x} \right)$$

$$\vec{x}' = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

x_1, x_2 are solutions

$x_1 + x_2$ is also a soln.

How we write our solution

$$\vec{x} = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2$$

General Solution

$$\vec{x}' = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} x \;,\; \vec{x}(0) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$\uparrow A$$

1) Find eigenvalues (λ's):

$$A^{\prime} = A - \lambda I$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{pmatrix}$$

$$\det|A'| = (ad - bc) = 0$$

$$(1-\lambda)(2-\lambda) - 6 = 0$$

$$2 - 2\lambda - \lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = 4$$

2) Find eigenvectors.

$$= \begin{pmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{pmatrix}$$

$\lambda_1 = -1$ ↙

$\lambda_2 = 4$ ↘

$$\begin{pmatrix} 1-(-1) & 2 \\ 3 & 2-(-1) \end{pmatrix} \qquad \begin{pmatrix} 1-4 & 2 \\ 3 & 2-4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} \qquad = \begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$2\eta_1 + 2\eta_2 = 0 \qquad -3\eta_1 + 2\eta_2 = 0$$

$$2\eta_1 = -2\eta_2 \qquad -3\eta_1 = -2\eta_2$$

$$\eta_1 = -\eta_2 \qquad \eta_1 = \frac{2}{3}\eta_2$$

set $\eta_2 = 1$

$$v_1 = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} -\eta_2 \\ \eta_2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}\eta_2 \\ \eta_2 \end{pmatrix}$$

set $\eta_2 = 3$

$$v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$G.S. = C_1 e^{-t}\begin{pmatrix}-1\\1\end{pmatrix} + C_2 e^{4t}\begin{pmatrix}2\\3\end{pmatrix}$$

$$\vec{x}(0) = \begin{pmatrix}0\\-4\end{pmatrix}$$

$$\begin{pmatrix}0\\-4\end{pmatrix} = C_1 e^{-0}\begin{pmatrix}-1\\1\end{pmatrix} + C_2 e^{4(0)}\begin{pmatrix}2\\3\end{pmatrix}$$

$$\begin{cases} 0 = C_1(-1) + C_2(2) \\ -4 = C_1(1) + C_2(3) \end{cases}$$

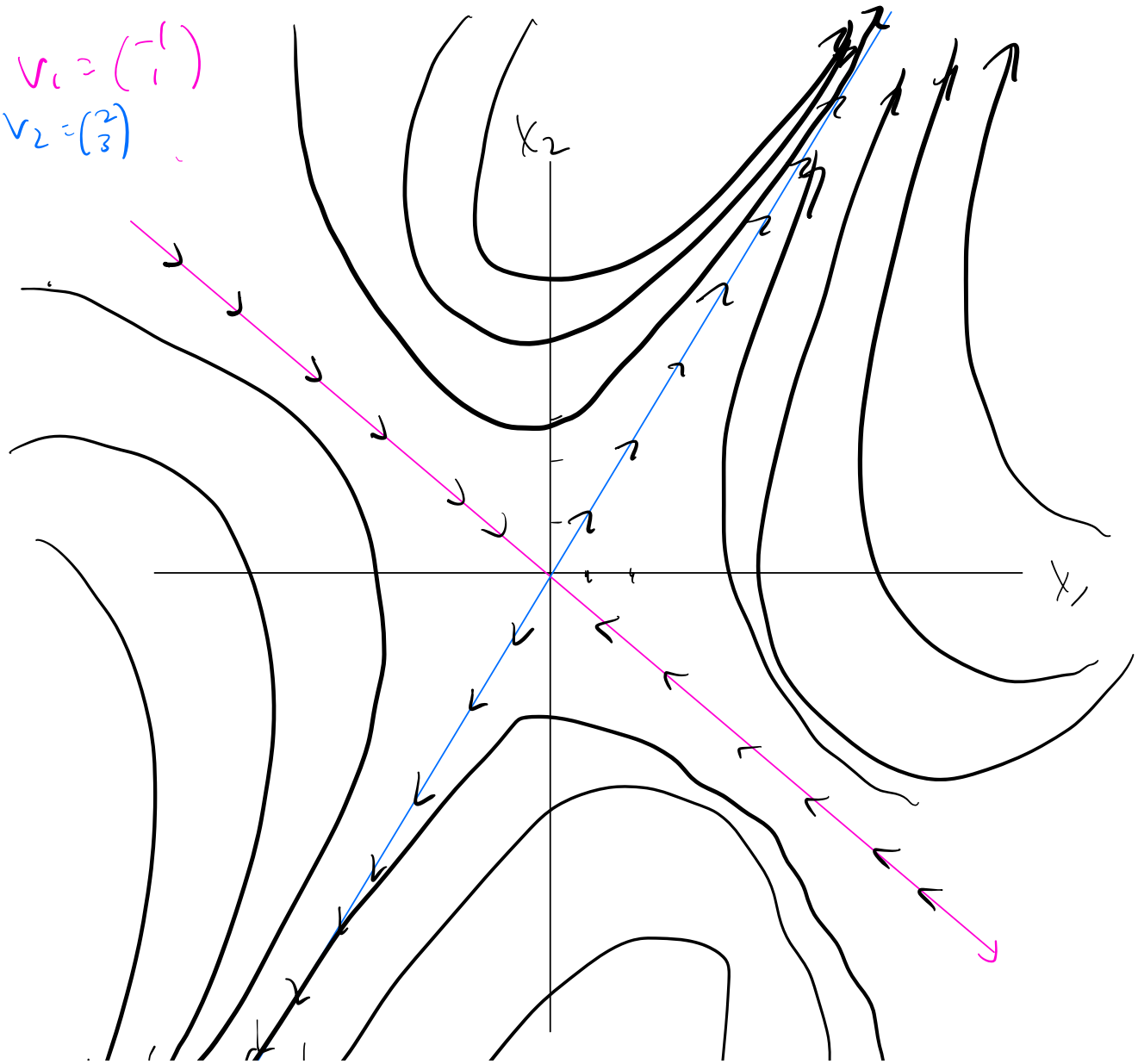
Solve the SoE.

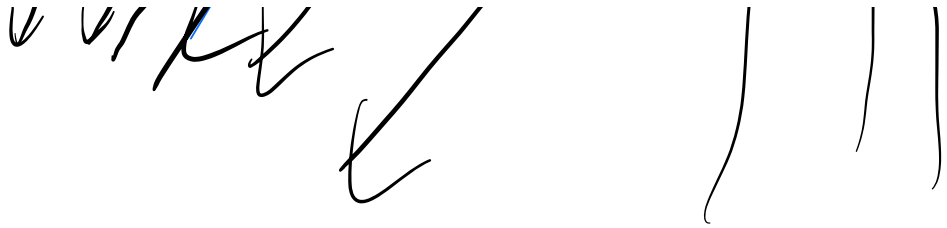
$$\begin{cases} C_1 = -1.6 \\ C_2 = -.8 \end{cases}$$

$$P.S. = -1.6e^{-t}\begin{pmatrix}-1\\1\end{pmatrix} - .8e^{4t}\begin{pmatrix}2\\3\end{pmatrix}$$

Next part:

Plot the phase plot:





One + eigenvalue

one − eigenvalue:

Saddle point