x, y, z

$$\left.\begin{aligned} \frac{dx}{dt} &= x + 4z \\ \frac{dy}{dt} &= 2y \\ \frac{dz}{dt} &= 3x + y - 3z \end{aligned}\right\} \quad A = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{pmatrix}$$

$$A - \lambda I$$

$$\begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & 0 & 4 \\ 0 & 2-\lambda & 0 \\ 3 & 1 & -3-\lambda \end{pmatrix}$$

$$1-\lambda\begin{pmatrix}2-\lambda & 0\\ 1 & -3-\lambda\end{pmatrix} - 0\begin{pmatrix}0 & 0\\ 3 & -3-\lambda\end{pmatrix} + 4\begin{pmatrix}0 & 2-\lambda\\ 3 & 1\end{pmatrix} = 0$$

$$(1-\lambda)\left[(2-\lambda)(-3-\lambda) - 0\right]$$

~~$- 0\left((0)(-3-\lambda) - (0)(1)\right)$~~

$$+ 4\left((0)(1) - (2-\lambda)(3)\right) = 0$$

$$(1-\lambda)\left[-6+3\lambda-2\lambda+\lambda^2\right]$$

$$+ 4\left[(0-6+3\lambda)\right] = 0$$

$$(1-\lambda)\left[\lambda^2+\lambda-6\right] + 4(3\lambda-6) = 0$$

$$\lambda^2+\lambda-6-\lambda^3-\lambda^2+6\lambda+12\lambda-24=0$$

$$-\lambda^3+19\lambda-30=0$$

$$\lambda = 2, 3, -5$$

$$A - \lambda I$$

$$\begin{pmatrix} 1-\lambda & 0 & 4 \\ 0 & 2-\lambda & 0 \\ 3 & 1 & -3-\lambda \end{pmatrix}$$

1) $$\begin{pmatrix} 1-2 & 0 & 4 \\ 0 & 2-2 & 0 \\ 3 & 1 & -3-2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 4 \\ 0 & 0 & 0 \\ 3 & 1 & -5 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 4 \\ 0 & 0 & 0 \\ 3 & 1 & -5 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = 0$$

$$-k_1 + 0k_2 + 4k_3 = 0$$

$$k_1 = 4k_3$$

$k_3 = 1$

$k_1 = 4$

$k_2 = -7$

$$3k_1 + 11k_2 - 5k_3 = 0$$

$$3(4k_3) + 11k_2 - 5k_3 = 0$$

$$7k_3 + k_2 = 0$$

$$k_2 = -7k_3$$

$$v_1 = \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix}$$

$$v_2 = \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix}$$

Repeat

$$v_3 = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$$

$$x = c_1 \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix} e^{2t}$$

$$+ c_2 \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix} e^{3t}$$

$$+ c_3 \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix} e^{-5t}$$

wkin?   6,3

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