1) $\int x^2 e^{-3x}\,dx$

$$\int fg' = fg - \int f'g$$

$f = x^2$ $\quad f' = 2x$

$g = \frac{-e^{-3x}}{3}$ $\quad g' = e^{-3x}$

$$\boxed{\frac{-x^2 e^{-3x}}{3} -} \int \frac{-2x\,e^{-3x}}{3}\,dx$$

$$\int \frac{-2x\,e^{-3x}}{3}\,dx$$

$$= -\frac{2}{3}\int x e^{-3x}\,dx$$

$$\int fg' = fg - \int f'g$$

$f = x \qquad f' = 1$

$g = -\frac{1}{3}e^{-3x} \qquad g' = e^{-3x}$

$$\frac{-xe^{-3x}}{3} - \int -\frac{1}{3}e^{-3x}\,dx$$

$$= \frac{-xe^{-3x}}{3} - \int \frac{-e^{-3x}}{3}\,dx$$

$u = -3x$

$du = -3\,dx$

$$= \frac{1}{9}\int e^{u}\,du$$

$$= \frac{e^{-3x}}{9}$$

$$\frac{-xe^{-3x}}{3} - \frac{e^{-3x}}{9}$$

$$-\frac{2}{3}\left(\frac{-xe^{-3x}}{3} - \frac{e^{-3x}}{9}\right)$$

$$\left[\frac{2xe^{-3x}}{9} + \frac{2e^{-3x}}{27}\right]$$

$$\boxed{\frac{-x^2e^{-3x}}{3} - \frac{2xe^{-3x}}{9} - \frac{2e^{-3x}}{27} + C}$$

$$\boxed{= -\frac{(9x^2+6x+2)e^{-3x}}{27} + C}$$

2) $\int \frac{\ln x}{x}\, dx$

$u = \ln x$, $du = \frac{1}{x}\, dx$

$dx = x\, du$

$$= \int \frac{u}{x}\, dx$$

$$= \int \frac{u}{x}\, x\, du$$

$$= \int u\, du$$

$$= \frac{u^2}{2}$$

$$= \frac{(\ln(x))^2}{2} + C = \boxed{\frac{\ln^2(x)}{2} + C}$$

3) $$\int \frac{3x + 2}{x^2 + x}\, dx$$

$$= \int \frac{3x + 2}{x(x+1)}$$

$$= \int \frac{A}{x} + \frac{B}{x+1}$$

$$= \int \frac{A(x)(x+1)}{x} + \frac{B(x)(x+1)}{(x+1)}$$

$$\underline{3x+2} = A(x+1) + B(x)$$

$x=-1 \rightarrow$

$$-1 = B(-1)$$

$$B=1$$

$x=0 \rightarrow$ $\quad 2 = A(1)$

$$A=2$$

$$= \int \left(\frac{1}{x+1} + \frac{2}{x} \right) dx$$

$$= \underbrace{\int \frac{1}{x+1}\, dx} + 2 \underbrace{\int \frac{1}{x}\, dx}_{\ln(x)}$$

$u = x+1$

$du = 1\, dx$

$dx = du$ $\quad \int \frac{1}{u}\, du$

$= \ln(u) = \underbrace{\ln(x+1) + 2\ln(x)}$

$$\boxed{= \ln(|x+1|) + 2\ln(|x|) + C}$$

4) $\int e^x \cos(x)\, dx$

$$\int fg' = fg - \int f'g$$

$f = \cos(x)$ $\qquad g = e^x$

$f' = -\sin(x)$ $\qquad g' = e^x$

$$= e^x \cos(x) - \int -e^x \sin(x)\, dx$$

$$\int fg' = fg - \int f'g$$

$f = -\sin(x)$ $\qquad g = e^x$

$f' = -\cos(x)$ $\qquad g' = e^x$

$$= e^{x}\cos(x) - \left(-e^{x}\sin(x) - \int -e^{x}\cos(x)\,dx\right)$$

$$= e^{x}\cos(x) - \left(-e^{x}\sin x + \int e^{x}\cos x\,dx\right)$$

$$L = e^{x}\cos(x) + e^{x}\sin x - L + C$$

$$2L = e^{x}\cos(x) + e^{x}\sin x + C$$

$$L = \boxed{\frac{e^{x}\cos(x) + e^{x}\sin(x)}{2} + C}$$

5) $$\int \frac{x^2 - 4x + 1}{(x^2+1)(x-1)^2}\,dx$$

$$= \int \frac{x^2+1}{(x^2+1)(x-1)^2} + \frac{(-4x)}{(x^2+1)(x-1)^2}\,dx$$

$\underline{2^{nd}}$ Part:

$$=4\int \frac{x}{(x-1)^2(x^2+1)}\,dx$$

$$\frac{x}{(x-1)^2(x^2+1)} \left\{ \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1} \right\}$$

$$x = (x-1)^2(Cx+D) + (x-1)(x^2+1)A + (x^2+1)B$$

$$x = x^3(A+C) + x^2(-A+B-2C+D) + x(A+C-2D)$$
$$-A+B+D$$

$$\left\{ \begin{array}{l} -A+B+D=0 \\ A+C=0 \\ -A+B-2C+D=0 \\ A+C-2D=1 \end{array} \right\}$$

$A = 0, \quad B = \frac{1}{2}, \quad C = 0, \quad D = \frac{1}{2}$

$$\frac{x}{(x-1)^2(x^2+1)} = \frac{0}{x-1} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{-\frac{1}{2}}{x^2+1} = \frac{\frac{1}{2}}{(x-1)^2} + \frac{-\frac{1}{2}}{x^2+1}$$

$$= \frac{1}{2(x-1)^2} - \frac{1}{2(x^2+1)}$$

$$= \int \frac{1}{2(x-1)^2} - \frac{1}{2(x^2+1)}$$

$$= \frac{1}{2} \int \frac{1}{(x-1)^2} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx$$

$\downarrow$ arctan(x)

$u = x-1 \quad du = 1$

$$\frac{1}{u^2} du = -\frac{1}{u}$$

$$= -\frac{1}{x-1}$$

$$= -\frac{\arctan(x)}{2} - \frac{1}{2(x-1)}$$

$$-4\left(-\frac{\arctan(x)}{2} - \frac{1}{2(x-1)}\right)$$

$$\left\{ 2\arctan(x) + \frac{2}{x-1} + C \right\}$$

1st Part:

$$\int \frac{x^2+1}{(x^2+1)(x-1)^2}\,dx$$

$$= \int \frac{1}{(x-1)^2}$$

$u = x-1$ $\qquad = \int \frac{1}{u^2}\,du$

$du = 1$

$$= -\frac{1}{u} = -\frac{1}{x-1}$$

$$= -\frac{1}{x-1} + C$$

Both:

$$= -\frac{1}{x-1} + C_1 +$$

$$2\arctan(x) + \frac{2}{x-1} + C_2$$

$$\boxed{= \frac{1}{x-1} + 2\arctan x + C_3}$$