March 13 2024

Laplace Transform

Inverse Laplace Transform

step functions, Convolution

---

improper integral

$$\int_0^\infty e^{ct}\,dt, \quad c \neq 0$$

$$= \lim_{n\to\infty} \int_0^n e^{ct}\,dt$$

$$= \lim_{n\to\infty} \left(\frac{1}{c} e^{ct}\right)\Big|_0^n$$

$$= \lim_{n\to\infty} \left(\frac{1}{c} e^{cn} - \frac{1}{c}\right)$$

LHS $\quad c \neq 0$

$= \text{inf} \quad c > 0$

$0 \quad c < 0$

$$\lim_{n\to\infty} \left(\frac{1}{c} e^{cn} - \frac{1}{c} \right)$$

$$\boxed{= -\frac{1}{c}, \quad c < 0}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

Ex

find $\mathcal{L}\{f(t)\}$, $f(t) = 1$

$$\int_0^\infty e^{-st}(1)\,dt = \boxed{\frac{1}{s},\quad s > 0}$$

$s = -c$

$\mathcal{L}\{f(t)\} = F(s)$

Linearity Property:

$$\mathcal{L}\{a f(t) + b\,g(t)\}$$

$$= aF(s) + b\,G(s).$$

$\mathcal{L}\{f(t)\}$ = not nice RHS

use table

$$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

$$\boxed{\mathcal{L}\{f(t)\} = 6\frac{1}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}}$$

inverse Laplace Transforms

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

Linearity:

$$\mathcal{L}^{-1}\{aF(s) + bG(s)\}$$

$$= a\mathcal{L}^{-1}\{F(s)\} + b\mathcal{L}^{-1}\{G(s)\}$$

Ex.

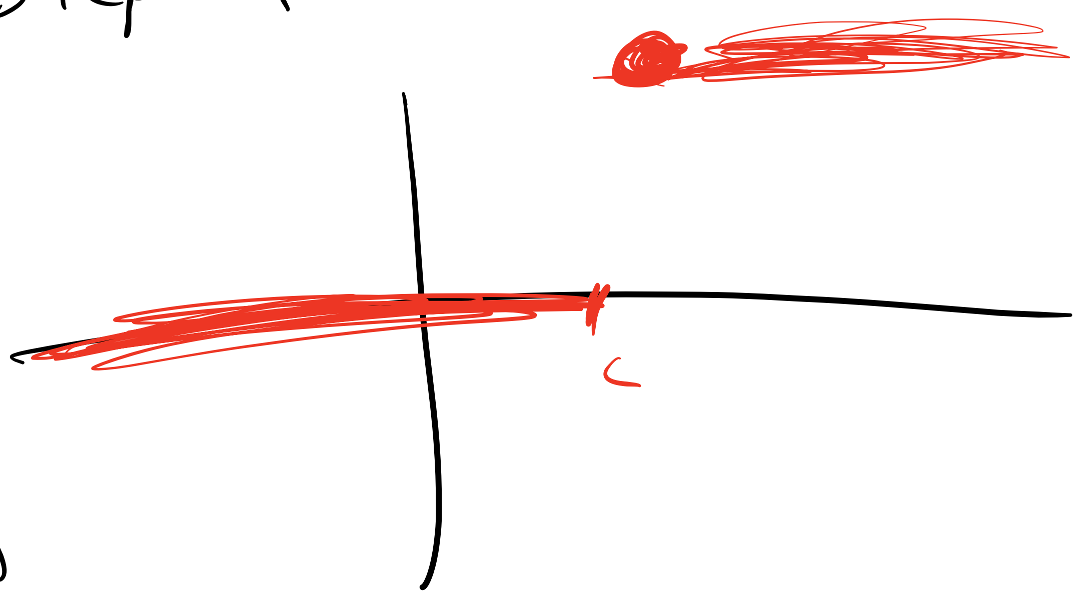
$$F(s) = \frac{6}{s} - \frac{1}{s-8} + \frac{4}{s-3}$$

$$\boxed{f(t) = 6 - e^{8t} + 4e^{3t}}$$

Ex.

$$G(s) = \frac{8}{3s^2+12} + \frac{3}{s^2-49}$$

Step functions



$u_c(t)$

$$= \begin{cases} 0 & \text{if } t < c \\ 1 & \text{if } t \geq c \end{cases}$$

$1 - u_c(t)$ opposite.

$$\left.\begin{array}{l} 3 - 3u_c(t) \\ 3(1 - u_c(t)) \end{array}\right\} \begin{array}{ll} 3 & \text{if } t < c \\ 0 & \text{if } t \geq c. \end{array}$$

Ex.

$$f(t) = \begin{cases} -4 & \text{if } t < 6 \\ 25 & \text{if } 6 \leq t < 8 \\ 16 & \text{if } 8 \leq t < 30 \\ 10 & \text{if } t \geq 30 \end{cases}$$

$$-4+(25-(-4))u_6(t)$$
$$+(16-25)u_8(t)$$
$$+(10-16)u_{30}(t)$$

as a step function.

$$\boxed{-4+29u_6(t)-9u_8(t)-6u_{30}(t)}$$

$$\mathcal{L}\{u_c(t)\}=\frac{e^{-cs}}{s}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-cs}}{s}\right\}=u_c(t)$$