

March 27, 2024

Laplace Transforms \rightarrow Differential Eqns.

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\{y'(t)\} = sY(s) - y(0)$$

$$\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

Ex. $y'' + 2y' + y = 5, \quad y(0) = 2, \quad y'(0) = 0$

$y = ?$

$$\mathcal{L}\{y'' + 2y' + y\} = \mathcal{L}\{5\}$$

$$s^2 \rightarrow s \dots Y(s) \dots = \frac{5}{s}$$

Solve for $Y(s)$:

$$Y(s) = \frac{\text{---}}{\text{---}}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

Ex.

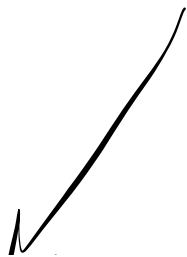
$$y' - y = e^{3t}, \quad y(0) = 2$$

$$sY(s) - y(0) - Y(s) = \frac{1}{s-3}$$

$$sY(s) - 2 - Y(s) = \frac{1}{s-3}$$

$$Y(s)(s-1) - 2 = \frac{1}{s-3}$$

$$Y(s) = \frac{1}{(s-3)(s-1)} + \frac{2}{s-1}$$



$$\frac{1}{(s-3)(s-1)} = \frac{A}{s-1} + \frac{B}{s-3}$$

$$1 = (s-3)A + (s-1)B$$

$$1 = As - 3A + Bs - 1B$$

$$1 = s(A+B) - 3A - B$$

$$A + B = 0$$

$$A = -B$$

$$-3A - B = 1$$

$$-3(-B) - B = 1$$

$$3B - B = 1$$

$$2B = 1$$

$$B = \frac{1}{2}$$

$$A = -\frac{1}{2}$$

$$= \frac{-\frac{1}{2}}{s-1} + \frac{\frac{1}{2}}{s-3}$$

$$= -\frac{1}{2} \left(\frac{1}{s-1} \right) + \frac{1}{2} \left(\frac{1}{s-3} \right)$$

$$Y(s) = -\frac{1}{2} \left(\frac{1}{s-1} \right) + \frac{1}{2} \left(\frac{1}{s-3} \right) + \frac{2}{s-1}$$

$$y(t) = -\frac{1}{2} e^t + \frac{1}{2} e^{3t} + 2e^t$$

$$y(t) = \frac{3}{2} e^t + \frac{1}{2} e^{3t}$$

Find the Laplace transform
of

$$(2t + 1) u_2(t).$$

$$\rightarrow f(t-c) u_c(t)$$

$$f(t-2) = 2t + 1 \checkmark$$

$$f(t) = 2t + 5$$

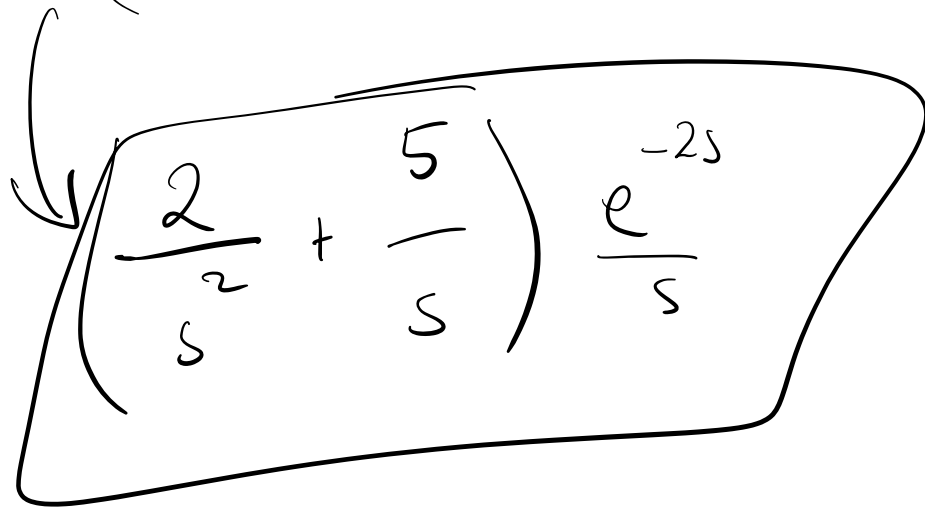
Alternatively

$$(2t + 1) u_2(t) \\ = (2(t-2+2) + 1) u_2(t)$$

$$(2(t-2) + 4(t+1)) u_2(t)$$

$$(2(t-2) + 5) u_2(t) = f(t-2) u_2(t)$$

$$(2t+5) u_2(t) = f(t) u_2(t)$$


$$\left(\frac{2}{s^2} + \frac{5}{s} \right) \frac{e^{-2s}}{s}$$

L^{-1}

$\{E_{2,1}\}$

$$y'' + 8y' + 2y = 0$$

$$y(0) = 1$$

$$y'(0) = 0,$$

Solve for $y(t) = ?$

$$Y(s) = \frac{s+8}{s^2+8s+2}$$

We'll stop there.

Ex. inverse Laplace

$$Y(s) = \frac{s+9}{s^2+6s+5}$$

$$y(t) = 2e^{-t} - e^{-5t}$$