4.6

$F = ma$

ma

mu''

$$mu'' = mg - k(L+u) - \gamma u' + F(t)$$

force due to gravity; force due to spring; damping force; ext. force

$$mu'' + \gamma u' + ku = mg - kL + F(t)$$

$$mg = kL$$

$$mu'' + \gamma u' + ku = F(t)$$

$u(0) = u_0$ in. position

$u'(0) = u_0'$ in. velocity

No external force, no damping.

$$mu'' + ku = 0$$

Solve in terms of u.

$$mr^2 + k = 0$$

$$r = \pm i\sqrt{\frac{k}{m}}$$

$$\sqrt{\frac{k}{m}} = \omega_0$$

"natural frequency"

$$r = \pm \omega_0 i$$

$$u(t) = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t)$$

Ex:

10 lb object stretching a spring 1/2 foot.

spring displaced 15 inches up

velocity of 1 ft/sec downward.

$$mu'' + ku = 0$$

$$m = \frac{10}{32} = \frac{5}{16}$$

$$mg = kl$$

$$k = \frac{mg}{L} = \frac{\left(\frac{5}{16}\right)(32)}{1/2} = 20$$

$$\frac{5}{16}u'' + 20u = 0$$

$$r = \pm i\sqrt{\frac{20}{(5/16)}}$$

$$r = \pm i\sqrt{64}$$

$r = \pm 8i$

$$u(t) = C_1 \cos(8t) + C_2 \sin(8t)$$

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1) Free ($F(t) = 0$), undamped

$$mu'' + ku = 0$$

Solve C.E. $\rightarrow$

have a solution.

Easy

Free ($F(t) = 0$), damped

$$mu'' + \gamma u' + ku = 0$$

Solve C.E.

$b^2 - 4ac$

$> 0 \rightarrow$ real roots

$< 0 \rightarrow$ complex roots

$= 0 \rightarrow$ repeated roots

2) Forced ($F(t) \neq 0$),

Undamped

Forced, ($F(t) \neq 0$)

damped

Use Undetermined Coeffs.

G.S. < Find C.S.
Find P.S.

G.S. = C.S + P.S.