03/06

Today:

Book: "Brannon pdf differential equations"

- Finish damping/oscillator stuff
- Variation of Parameters
  2 methods
- Talk about quiz
  - RLC-series (4.4)
  - Undetermined Coeff. (4.5)
  - Oscillators (4.6)

Free, undamped oscillator: $mu'' + 0u' + ku = 0$

free, damped oscillator: $mu'' + \gamma u' + ku = 0$

Free

Forced, undamped oscillator

forced, damped oscillator

RHS = $F(t)$

Gen. Solution = C.S. + P.S.

Forcing function

if $\quad RHS = F_0 \cos(\omega t)$ $\qquad 2\cos(5t)$

or

$F_0 \sin(\omega t)$

$A t\cos(5t) + Bt\sin(5t)$

ex. $mu'' + \gamma u' + ku = F_0 \cos(\omega t)$

$$Y_p = A\cos(\omega t) + B\sin(\omega t)$$

$$y_c = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t) \qquad C_1 \cos(5t) + C_2 \sin(5t)$$

$\omega_0 \neq \omega$

Then $\quad y_p = \dfrac{F_0}{k - m\omega^2} \cos(\omega t)$

$\underline{\omega_0 = \omega}$ Then $\quad y_p = \dfrac{F_0}{2m\omega_0} t\sin(\omega_0 t)$

$$y_p = \frac{2}{2(m)5} t\sin(5t)$$

Then C.S. + P.S. = G.S. 4.6

y_1, y_2 = y_c solutions $g(t)$ = non-homogeneous term

4.7 Variation of Parameters

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$y_p = -y_1 \int \frac{y_2\, g(t)}{W(y_1, y_2)}\,dt + y_2 \int \frac{y_1\, g(t)}{W(y_1, y_2)}\,dt$$

$$y'' + 9y = 3\tan(3t)$$

$$r^2 + 9 = 0$$

$$r^2 = -9$$

$r = \pm 3i$

$y_c = C_1 \cos(3t) + C_2 \sin(3t)$

$y_1 = \cos(3t)$

$y_2 = \sin(3t)$

$$W = \begin{vmatrix} \cos(3t) & \sin(3t) \\ -3\sin(3t) & 3\cos(3t) \end{vmatrix} = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= 3\cos^2(3t) + 3\sin^2(3t) = 3$$

$$y_p = -y_1 \int \frac{y_2\, g(t)}{W(y_1, y_2)}\,dt + y_2 \int \frac{y_1\, g(t)}{W(y_1, y_2)}\,dt$$

$$y_p = -\cos(3t)\int \frac{3\sin(3t)\tan(3t)}{3}\,dt$$

$$+\sin(3t)\int \frac{3\cos(3t)\tan(3t)}{3}\,dt$$

$$y_p = -\cos(3t)\int \frac{\sin^2(3t)}{\cos(3t)}\,dt + \sin(3t)\int \sin(3t)\,dt$$

$$= \frac{1}{\cos(3t)} - \frac{\cos^2(3t)}{\cos(3t)}$$

$$= -\cos(3t)\int \frac{1-\cos^2(3t)}{\cos(3t)}\,dt + \sin(3t)\int \sin(3t)\,dt$$

$$\boxed{= -\frac{1}{3}\cos(3t)\left(\left[\ln\left(\sec(3t)+\tan(3t)\right)\right] - \sin(3t)\right) + \frac{\sin(3t)}{3}\left(-\cos(3t)\right)}$$

$y_p =$

$$\boxed{y_g = C_1\cos(3t) + C_2\sin(3t) + y_p}$$

$$x' = Ax + g(t)$$

$$X' = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} X + \begin{pmatrix} 18 \\ 3t \end{pmatrix}$$

$C.E._{2\times 2\ A} = \lambda^2 - Tr(A)\lambda + det(A)$

$$C.S. \begin{cases} X_1(t) = e^{-t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ X_2(t) = e^{t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{cases}$$

Use alternate VoP formula:

$$X_p = X(t)\int X^{-1}(t)\, g(t)\, dt$$

$$X(t) = \begin{bmatrix} e^{-t} & e^{t} \\ 0 & 2e^{t} \end{bmatrix}$$

$$X^{-1}(t) = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 2e^{t} & -e^{t} \\ 0 & e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} e^{t} & \frac{-e^{t}}{2} \\ 0 & \frac{e^{-t}}{2} \end{bmatrix}$$

$$X\int X^{-1}(t)g(t)$$

$$\int X^{-1}(t)g(t) = \begin{bmatrix} e^{t} & -\frac{e^{t}}{2} \\ 0 & \frac{e^{-t}}{2} \end{bmatrix}\begin{bmatrix} 18 \\ 3t \end{bmatrix}$$

$$\int X^{-1}(t)g(t) = \begin{bmatrix} \frac{33}{2}e^{t} + \frac{3}{2}te^{t} \\ -\frac{3}{2}e^{-t} - \frac{3}{2}te^{-t} \end{bmatrix}$$

$$X\int X^{-1}(t)g(t) = \begin{bmatrix} 18+3t \\ -3-3t \end{bmatrix} = y_p$$

$$\boxed{G.S = C_1e^{-t}\begin{pmatrix}1\\0\end{pmatrix} + C_2e^{t}\begin{pmatrix}-1\\2\end{pmatrix} + \begin{bmatrix}18+3t\\-3-3t\end{bmatrix}}$$

a) Wksht 4.7

$$x' = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 18 \\ 3t \end{pmatrix}$$

might be better to

use inverse matrix method

b) $y'' + 2y' + y = 3e^{-t}$

Could

1) use $-y_1 \int \frac{y_2 g(t)}{W} + y_2 \int \frac{y_1 g(t)}{W}$

2) set up as a system like (1), and solve using inverse matrix method

Ex.

$$y'' + 16y = 2\sec(4t),$$

$$-\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

Solve the differential eqn.

$$\text{C.S} = c_1 \cos(4t) + c_2 \sin(4t)$$

$$\phi = \begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix} = \begin{bmatrix} \cos(4t) & \sin(4t) \\ -4\sin(4t) & 4\cos(4t) \end{bmatrix}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$u_1 = \int - \frac{y_2 g(t)}{|\phi|}$$

$$u_2 = \int \frac{y_1 g(t)}{|\phi|}$$

$$u_1 = -\int \frac{2\sin(4t)\sec(4t)}{4}$$

$$u_2 = \int \frac{2}{4} dt$$

$$u_1 = -\frac{1}{2}\int \tan(4t)\,dt$$

$$u_2 = \frac{t}{2}$$

$$u_1 = -\frac{\ln|\sec(4t)|}{8}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_1 = \cos(4t)$$

$$y_2 = \sin(4t)$$

$$\boxed{y_p = -\cos(4t)\ln(\sec 4t) + t\sin(4t)}$$

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