1) $y'' + 36y = 0$

$r^2 + 36 = 0$

$r^2 = -36$

$r = \pm 6i$

$y = C_1 \cos(6t) + C_2 \sin(6t).$

2) $y(0) = 6$

$y'(0) = -6$

$y' = -6C_1 \sin(6t) + 6C_2 \cos(6t)$

$6 = C_1 \cos(0) + C_2 \sin(0)$

$-6 = -6C_1 \sin(0) + 6C_2 \cos(0)$

$$6 = C_1 + 0$$

$$-6 = -6C_1(0) + C_2$$

$$C_1 = 6$$

$$C_2 = -1$$

$$y = 6\cos(6t) - 1\sin(6t)$$

3) $y'' + y' - 6y = 0$

$\downarrow$

$r^2 + r - 6 = 0$

$(r-2)(r+3)$

roots: $2, -3$

Soln: $C_1 e^{2t} + C_2 e^{-3t}$

4) Cauchy - Euler Equation

Homogenous second-order equation

1) $ax^2y'' + bxy' + cy = 0$, $x > 0$

when $a \neq 0$, $b, c \rightarrow$ constants

$t = \ln(x)$, chain rule

(2) $a\dfrac{d^2y}{dt^2} + (b-a)\dfrac{dy}{dt} + cy = 0.$

a 4) Solve diff eq.

$x^2y'' + xy' + 4y = 0$ $\quad t = \ln(x)$

$a = 1 \quad b = 1 \quad c = 4$

$ay_t'' + (b-a)y_t' + cy_t = 0$

$y_t'' + 4y_t = 0.$

$r^2 + 4 = 0 \Rightarrow r^2 = -4$

$r = \pm 2i$

$y_c = C_1\cos(2t) + C_2\sin(2t)$

$$y_c(x) = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x).$$

Cauchy - Euler equations.