Quiz Review

10/26/2023

1) Variation of Parameters
2) Undetermined Coefficients
3) Electrical / Mechanical Problem

VoP:

$$y'' + p(x)y' + H(x)y = G(x),$$

$$y_1(x) \qquad y_2(x)$$

$$G.S. = C.S. + P.S.$$

$$= C_1 y_1 + C_2 y_2$$

$$= -y_1 \int \frac{y_2 g(t)}{W(y_1, y_2)} + y_2 \int \frac{y_1 g(t)}{W(y_1, y_2)}$$

Ex 1:

$$y'' - \left(1 + \frac{1}{t}\right)y' + \frac{1}{t}y = t,$$

$$y_1(t) = e^t \qquad y_2(t) = t + 1$$

$$W = \det \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \det \begin{vmatrix} e^t & t+1 \\ e^t & 1 \end{vmatrix}$$

$$= e^t - e^t(t+1)$$

$$P.S. = e^t \int \frac{(t+1)t}{-te^t}\,dt + (t+1)\int \frac{e^t t}{te^t}\,dt$$

$$= e^t\left(e^{-t}(t+2)\right) - (t+1)t$$

$$= t^2 - 2t - 2$$

$$C.S. = C_1 e^t + C_2(t+1)$$

G.S.: $C_1 e^t + C_2(t+1) - t^2 - 2t - 2$

Ex. 2 (2×2) Wkst 4.7

$$X' = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} X + \begin{pmatrix} 1 \\ 3t \end{pmatrix}$$

$$X_1(t) = e^{-t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$X_2(t) = e^{t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

We can use

$$X_p = X(t)\int X^{-1}(t)\, g(t)\, dt$$

$$X(t) = \begin{bmatrix} e^{-t} & -e^{t} \\ 0 & 2e^{t} \end{bmatrix}$$

$$X^{-1}(t) = \begin{bmatrix} e^t & te^t \\ 0 & \frac{1}{2}e^{-t} \end{bmatrix}$$

$$g(t) = \begin{pmatrix} 18 \\ 3t \end{pmatrix}$$

$$\int X^{-1}(t) g(t) = \begin{bmatrix} \frac{33}{2}e^t + \frac{3}{2}te^t \\ -\frac{3}{2}e^{-t} - \frac{3}{2}te^{-t} \end{bmatrix}$$

$$X_p(t) = \underbrace{X(t)}_{2\times 2} \int X^{-1}(t) g(t) = \begin{bmatrix} 18+3t \\ -3-3t \end{bmatrix}$$

$$\text{G.S.} = C_1 e^{-t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{t}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$G.S. = C_1 e^{-t}\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{t}\begin{pmatrix}-1\\2\end{pmatrix} + \begin{bmatrix}18+3t\\-3-3t\end{bmatrix}$$

Underdetermined Coefficients

initial guess

	RHS	Guess
(1)	e^{cx}	Ae^{cx}
(2)	$\sin(\omega)$	$A\cos(\gamma) + B\sin(\gamma)$
	$(1)+(2)$	$(1)_G + (2)_G$
	$(1)(2)$	$(1)_G(2)_G$

Ex. 3 $y'' + 3y' + 2y = e^{3t}$

$y(0) = 0, \quad y'(0) = 0$

Fundamental Set: $\{e^{-t}, e^{-2t}\}$

$P.S. = Ae^{3t} = y_p(1)$

$3Ae^{3t} = y'_p(1)$

$9Ae^{3t} = y''_p(1)$

$9Ae^{3t} + 9Ae^{3t} + 2Ae^{3t} = e^{3t}$

$20Ae^{3t} = e^{3t}$

$A = \frac{1}{20}$

$G.S. = y = C_1e^{-t} + C_2e^{-2t} + \frac{1}{20}e^{3t}$

If the fundamental solutions aren't given:

If you get sth like this →

$$y'' - 2y' - 3y = 3e^{2t}, \quad y(0) = 0, \quad y'(0) = 1$$

↓ you can solve for your solutions:

$$\lambda^2 - 2\lambda - 3 = 0$$

$$(\lambda - 3)(\lambda + 1) = 0$$

$$\lambda = 3, -1$$

$$C.S. = C_1 e^{3t} + C_2 e^{-t}$$

Then, guess Ae^{2t}:

$$y_p(t) = Ae^{2t}$$

and finish the problem like in Ex. 3.

Electrical Mechanical

4. Find charge on capacitor on LRC-series circuit.

$L = 0.05$ H

$R = 1$ ohm

$C = 0.04$ Farads

$E(t) = 0$ Volts

$q(0) = 7$ Coulombs

$I(0) = 0$ amperes

$\downarrow$

$q'(0) = 0$

$$Lq'' + Rq' + \frac{q}{C} = E(t)$$

$$0.05q'' + q' + \frac{q}{0.04} = 0$$

$$q'' + 20q + 500q = 0$$

$$\lambda^2 + 20\lambda + 500 = 0$$

$$\lambda = \frac{-20 \pm \sqrt{400 - 2000}}{2}$$

$$\lambda = -10 \pm 20i$$

G.S. =

$$q(t) = e^{-10t}\left(C_1\cos(20t) + C_2\sin(20t)\right)$$

A) find the charge at

$t = 0.02$

B) Determine first time that charge = 0.

$t = 0 \rightarrow q = 7$

$7 = e^{0}\left(C_1\cos(20(0)) + C_2\sin(20(0))\right)$

$\boxed{7 = C_1}$

$$q'(t) = (-10)e^{-10t}\left(C_1\cos(20t) + C_2\sin(20t)\right)$$
$$- 20e^{-10t}\left(C_1\sin(20t) - C_2\cos(20t)\right)$$

$$0 = -10C_1 + 20C_2$$

$$70 = 20C_2$$

$$\boxed{C_2 = \frac{7}{2}}$$

$$= -10 \pm 20i, \quad C_1 = 7, \quad C_2 = \frac{7}{2}$$

$$q(t) = e^{-10t}\left(7\cos(20t) + \frac{7}{2}\sin(20t)\right)$$

A: $q(0.02) =$

$$e^{-0.2}\left(7\cos(0.4) + \frac{7}{2}\sin(0.4)\right)C$$

B:

$$0 = e^{-10t}\left(7\cos(20t) + \frac{7}{2}\sin(20t)\right)$$

$$0 = 7\cos(20t) + \frac{7}{2}\sin(20t)$$

$$0 = \frac{7\cos(20t) + \frac{7}{2}\sin(20t)}{\cos(20t)}$$

$$0 = 7 + \frac{7}{2}\tan(20t)$$

$$-7 = \frac{7}{2}\tan(20t)$$

$$-2 = \tan(20t)$$

$$t = 0.1017$$