1) Separation of Variables

2) Integrating Factor

3) order, linear / nonlinear,
phase line (direction field)

---

1)

$$\frac{dy}{dx} = 2y^2 e^{4x}$$

$$y(0) = -1$$

$$\frac{dy}{dx} = 2y^2 e^{4x}$$

$$\frac{dy}{2y^2} = e^{4x}\,dx$$

$$\frac{1}{2}\,y^{-2}\,dy = e^{4x}\,dx$$

$$\frac{1}{2}\left[-y^{-1}\right] = \frac{1}{4}e^{4x} + C$$

$$\int y^{-2}\,dy = \frac{y^{-2+1}}{-2+1} + C$$

$$-\frac{1}{2}\,y^{-1} = \frac{1}{4}e^{4x} + C$$

$$y^{-1} = -\frac{1}{2}e^{4x} + C$$

$$y = -\left(\frac{1}{2}e^{4x} + C\right)^{-1}$$

General Solution

$$x = 0, \quad y = -1$$

$$-1 = -\left(\frac{1}{2}e^{(0)} + C\right)^{-1}$$

$$-1 = -\left(\frac{1}{2} + C\right)^{-1}$$

$$-1 = \frac{1}{-\left(\frac{1}{2} + C\right)}$$

$$\frac{1}{2} + C = 1$$

$$C = \frac{1}{2}$$

$$y = -\left(\frac{1}{2}e^{4x} + \frac{1}{2}\right)^{-1}$$

Final Answer

2) $\frac{dy}{dx} + 2xy = (x+2)e^{-x^2}$

$$y' + P(x)y = H(x)$$

I.F.

$$\mu(x) = e^{\int P(x)\,dx}$$

$$= \int 2x\, dx \qquad x^2$$

$$e \qquad = e$$

$$e^{x^2} y' + e^{x^2} 2xy = \cancel{e^{x^2}}(x+2)\cancel{e^{-x^2}}$$

$$\int \left(e^{x^2} y' * e^{x^2} 2xy\right) = \int x+2$$

$$\int \left(e^{x^2} y\right)' = \frac{1}{2}x^2 + 2x + C$$

$$(u'y)'$$

$$e^{x^2} y = \frac{1}{2} x^2 + 2x + C$$

$$y = \frac{.5x^2 + 2x + C}{e^{x^2}}$$

IVP. $y(0) = 1$

$$1 = \frac{.5(0) + 2(0) + C}{e^0}$$

$$1 = \frac{0 + 0 + C}{1} \qquad C = 1$$

$$y = \frac{.5x^2 + 2x + 1}{e^{x^2}}$$

3) Consider the DE:

$$\frac{d^3y}{dx^3} + y^2\frac{dy}{dx} + y = x + 2$$

a) order? 3

b) Linear / NL? Non-linear

Linear:

$a y + b y' + c y'' + d y''' + \dots$

$a = 1$, $f(x), \dots$ ~~$f(y)$~~

~~$f(x,y)$~~

4) $y' = y^2 - 2$

a) Find the eq. points.

b) Phase line.

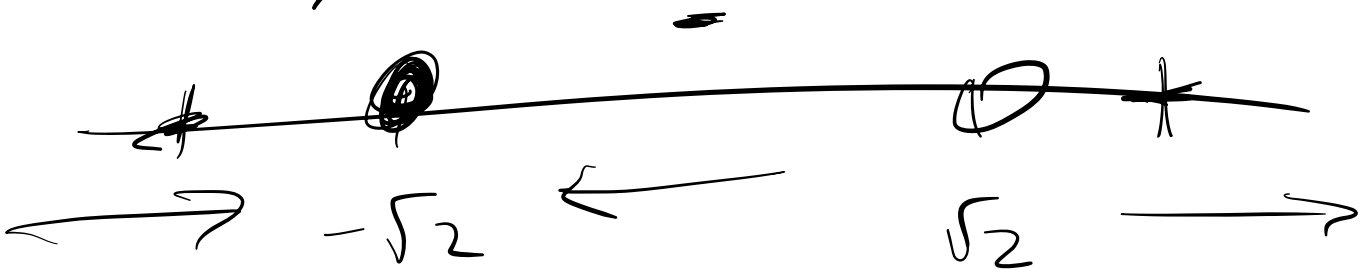
a) $y' = 0$

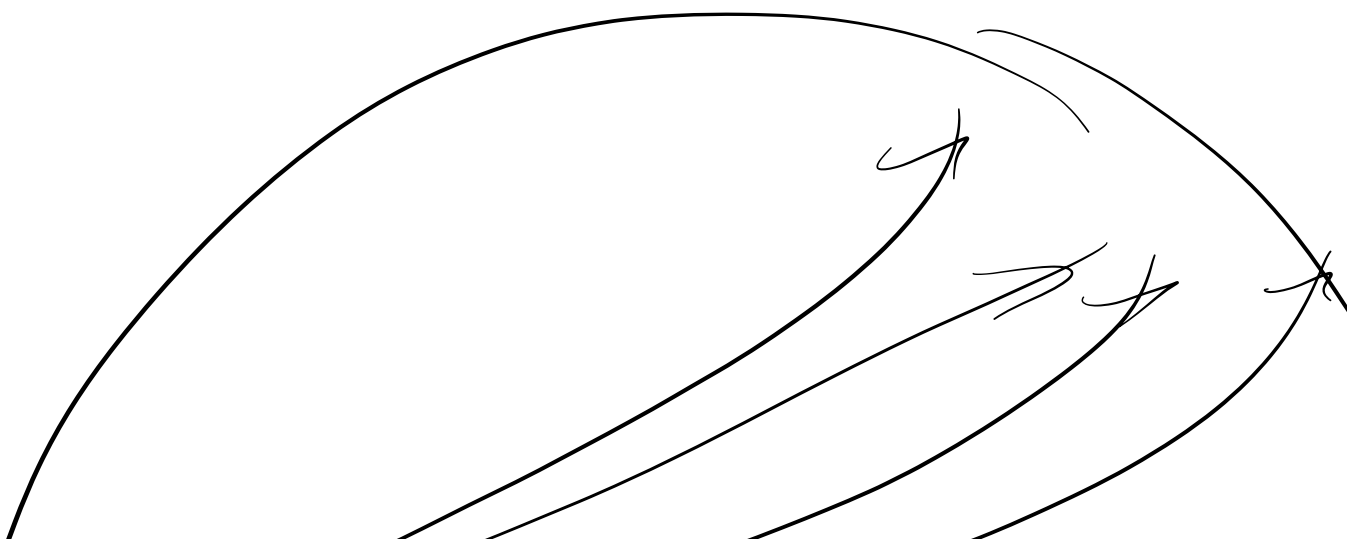
$y^2 - 2 = 0$

$y^2 = 2$

$y = \sqrt{2}, -\sqrt{2}$

b)





Office Hours

Tuesday 1-2 PM

Clough 280

(Math Lab)